Addressing large cohorts of first year mathematics students in lectures

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We investigate university teaching practices in the context of lectures to identify how students' learning needs are conceptualized and addressed in this context. In this paper we focus on one lecturer's goals for teaching and the associated teaching practices. His teaching to a large cohort of mathematics students in a Calculus course is analysed by using grounded techniques and the Teaching Triad construct (Jaworski, 1994). The analysis suggests that this lecturer's main goal is to help students start their university studies smoothly. In his practice he tries to support students with the advanced mathematical content to be learned and to introduce them to aspects of advanced mathematical thinking. The Triad brings to our insight that Sensitivity to Students could be central in teaching, even in the lecture context.

Keywords: teaching practices, lectures, Sensitivity to Students.

INTRODUCTION

Lectures have been widely criticized as a method of teaching but remain the common element of teaching mathematics at the university level with the potential to contribute significantly to learning (Pritchard, 2015). Despite of being the predominant format of teaching at university level, the lecture format has attracted very few studies possibly because the lecture is taken as a description of how teaching practice looks like at this level (Speer, Smith and Horvath, 2010). However, existed research studies in mathematics education have shown that teaching in lectures may vary and needs studying (e.g. Weber, 2004). Studying teaching practices in lectures, especially those practices that afford learning potentials to students, could be an important source of insights into the processes and practices of university mathematics teaching. Such studying could contribute to researchers' awareness about potentials of university mathematics lecturing. It could also contribute to university lecturers' reflective thinking on their own practices towards the development of enriched learning opportunities for mathematics students.

A general question that we try to address is how teaching at this level, and in the particular context of lectures to large cohorts of students, takes students into account. There is a body of research seeking to characterize elements of teaching practice that takes students into account largely at school level (e.g. Stein, Engle, Smith and Hughes, 2008). However, university students, like the students at the other levels, have also learning needs particularly in the first year of their studies. For example, they struggle with the abstraction and formalism of university mathematics (Nardi 1996) and they experience difficulties related to the secondary-tertiary transition (Pritchard, 2015). It is essential to know how students' learning needs are conceptualized by university teachers forming goals for teaching and how these conceptualizations are enacted with specific teaching practices. Thus, we seek to address the scarcity of empirical research and to gain better understandings of the mathematics teaching at this level drawing on direct observations of teaching practice. In this paper we investigate the teaching practice of one lecturer who teaches Calculus in a mathematics department. He is a lecturer whom students seem to consider of great help and who has very high rates of students' success in the course's examinations. In particular, here: a) we identify this lecturer's goal-directed teaching practices related to students and b) we interpret the identified teaching practices in the particular context of lectures.

THE THEORETICAL BACKGROUND

Our theoretical perspective of teaching is that it is an activity which: "first, it aims to bring about learning, second, it takes account of where the learner is at, and, third it has regard for the nature of what has to be learnt" (Pring, 2000; p. 23). We employ the language of Activity Theory in relation to teaching actions and goals (Leontiev, 1978). Our perspective towards teaching practice is sociocultural; within this perspective we analyze our data and we interpret our findings in the social setting of a university mathematics amphitheatre and in the culture of mathematics. We agree with Morgan (2014), that the study of a university teacher's conscious goal-directed teaching actions makes more sense when these actions are interpreted in the light of the broader context within which this individual teacher is situated. The students in this context have, like every other student affective, social and cognitive 'needs' (this term is elaborated in Hannula, 2006). In fact, they move from the school culture which is organised around the mastery of rather familiar tasks to a culture where the routinization of practices is much more difficult (Artigue, Batanero and Kent, 2007). This 'move' could be eased in lectures according to Pritchard (2015) who argues that lecturers can help first year students deal with transition related challenges by paying attention to students' technical difficulties; by demonstrating how mathematicians think and how real mathematics are; and by giving mathematics a human face.

We investigate how students are taken into account in lectures responding to the calls for attention of "how and why teaching happens in certain ways" at university level (Speer Smith & Horvath, 2010). We adopt Speer et al.'s (2010) distinction between instructional activities and teaching practice. According to this distinction the lecture, the context of our study, is an instructional activity while teaching practice concerns what teachers do when they are planning, teaching and reflecting on their lesson. By teaching practice we mean the lecturer's teaching actions (what he does intentionally) and the rationale behind these actions.

We draw on studies that characterized teaching approaches through observations of practice at both secondary and university level. At the university level for example, Weber (2004) studied the teaching of one mathematician in a proof-oriented course and described his actions which influenced the way that his students attempted to learn the material. Mali, Biza and Jaworski (2014) identified characteristics of university mathematics teaching in the tutorial setting. At the secondary level, Lobato, Clarke and Ellis (2005) examined the processes of teachers' 'telling' and pointed out that telling is instructionally important since students cannot be expected to reinvent entire bodies of mathematics. Identifying levels of 'scaffolding' teaching practices that can enhance mathematics learning, Anghileri (2006) considered "explaining the ideas to be learned" as a central practice even if it is not so responsive to the learner. 'Explaining' is a code that we also use in our analysis. Moreover, Baxter and Williams (2010) addressed the "dilemma of telling" students what they need to know and facilitating their mathematical understandings at the same time while Grandi and Rowland (2013) pointed out the importance of the context in the management of the same dilemma. Drageset (2014) characterized in detail elements of teaching practice such as teachers' comments. The above studies focused on the teaching of one or a very small number of teachers and used qualitative approaches to categorize teaching actions and teaching approaches. Their findings informed the coding process in our attempt to identify the lecturer's actions and practices.

Our research tool in the endeavour to interpret the identified teaching practices in the context of the lectures is Jaworski's (1994) Teaching Triad (TT). TT is an analytic framework that emerged from an ethnographic study at secondary level. Its main goal was to capture essential elements of the complexity of mathematics teaching. Jaworski describes that the Triad consists of three "domains" of activity in which teachers engage: management of learning (ML), sensitivity to students (SS) and mathematical challenge (MC). ML describes how the teacher organizes the classroom learning environment. SS describes teacher's knowledge of students needs. SS has been shown to relate to both affective (e.g., offering praise) (SSA) and cognitive (e.g., inviting explanation) (SSC) domains. MC describes the challenges offered to students to engender mathematical thinking. The above elements are closely interrelated. Jaworski and Potari (2009) further pointed out to a need for a broader appreciation from the side of the teacher of what is possible for the students or how much help they might need to achieve teaching objectives; an appreciation which is not specifically related to particular students. They used the term "social sensitivity" to describe this dimension. The Triad has also been used in studying interactions in university mathematics tutorials (Nardi, Jaworski and Hegedus, 2005) but it has not been used in studying lecturing so far. It is a question for example, what is the potential meaning that Sensitivity to Students could gain in this setting.

METHODOLOGICAL ISSUES: DATA AND ANALYSIS

This paper is a part of an ongoing study with aim to investigate first year's university teaching in Greek mathematics departments. The topic in focus is Calculus, a compulsory first year course, is taught exclusively in a lecture format. Calculus is a topic also taught in high school (age 17). The main difference between Calculus taught in school and Calculus taught in university is in emphasis given to the concepts. In mathematics departments, Calculus courses have a more theoretical focus while in high school the emphasis is on computations and methods. The participant in the study presented here is a very active research mathematician and an experienced university teacher. In his department, the Calculus course is taught in two parts. The first part includes sequences of real numbers, functions and derivatives. The second part, from which is our data here, is taught during the spring semester and includes series of real numbers, integrals, sequences of functions and power series. The course is taught for 13 weeks, 6 hours per week (4 hours for theory and 2 hours for exercises), to large cohorts of students. While the Calculus course is compulsory, the attendance of the lectures is not. This means that a student can participate in the final exams even if she has not attended the lectures. The course is taught in parallel in three classes from three lecturers. There is an indicative alphabetical allocation of students in these three classes, which is proposed by the department, but, in practice, each student can attend whomever of the lecturers she chooses. Interestingly, the vast majority of students (200+) choose and attend this lecturer's class. Notably, first year Calculus is one of the most difficult courses for the students in this department and many students fail in the final exams. This failure leads students to take their degree in 6.5 years on average instead of 4 years which is the formal duration of studies for a mathematics degree in this department. The lecturer is aware of and interested in this problem. He keeps statistical information about students' success in Calculus courses. The rate of success of students in his course is very high. The course is supported by an accessible to all students web site (e-class) which includes general information about the course, notes and questions from past exams. Data for this lecturer were collected during two years (2012-2013) through lectures' observations (19 hours of lectures); field notes; and interviews right after some lectures discussing issues from teaching (7 interviews, conducted by the first author). In addition to interviews, informal short discussions with students during the time of collecting observational data were also conducted. All lectures and interviews were audio-recorded and transcribed.

In data analysis, grounded approaches (Charmaz, 2006) and the Teaching Triad (Jaworski, 1994) were used. The analysis was conducted in three layers. In the first one, each lecture was divided into episodes typically including a section where one theorem or one proposition was taught. Since the course was proof-based, the episodes included the largest part of the lectures. In each episode, teaching actions were coded. Grounded coding of lecturers' typical teaching actions (mostly observations from the lectures) as well as codes from the literature (e.g. Anghileri, 2006; Drageset, 2014) used to characterize teaching practices. In the second layer of analysis, the rationale of the teaching had been investigated through the analysis of the interviews (also divided in themes). Considering successively and thoroughly the outcomes of each of the first two phases of analysis resulted to the identification of the lecturer's teaching practices (repeated teaching actions and the rationale behind these actions). In the third layer of analysis, the TT was used as an analytical frame to gain insights into the nature of the identified goals and teaching practices. In this way we explored potentials of TT's elements at this level.

RESULTS

The lecturer seems to take into account the broader context into which teaching is situated. In particular, he considers that university newcomers face difficulties in their transition to university. Some of these difficulties relate to the advanced mathematical subject per se while some others relate to the new setting, for instance to the “enormous number of students” in lectures. The lecturer considers that these difficulties may lead some students to fail in the final examination and have a delay in their studies. He values that “it is important for students not to waste time in getting their degree. I know that they get lost in their first year studies”. Taking into account all these difficulties that students may have was judged as an expression of lecturer’s sensitivity to students (SS). As it is emerged from his first interview, his main goal is to help students to overcome these difficulties that is, as he says, to start their university studies “smoothly”.

> “We want as many students as possible to start their studies smoothly. Given the enormous number of students, general adaption difficulties of first year students and the difficulty of the subject, ideally the average student could pass all the compulsory courses in a time period of three years instead of two which is expected. I believe it is possible.”

He thinks that a good organization of the course is important for his goal.

> “Which are the needs for the course...? Students want to know what exactly is expected for them. Even these organizational things about the intermediate assessment etc… They want to know. This is one need for them: to be organized”.

He also takes into account that there are students who do not attend the lectures perhaps because they have to work in parallel with their studies for financial reasons. This may be an expression of lecturer’s sensitivity to students’ social background, a social sensitivity to students (designated as SSS from now on). SSS was identified also in this lecturer’s actual teaching (exemplified bellow).

> “When you believe that students attend the lectures, then you ignore all these students who do not attend and study and… You have also to think about a crowd of people who do not come here so, you have to take this into account.”

To support students who cannot attend the lectures, he assigns to a student to keep notes from the lectures, he corrects these notes once a week and he uploads the notes on e-class. Organizing the course and using the e-class was judged as managerial of students’ learning (ML) but also as an indication of social sensitivity (SSS).

In class, his main goal is carried out with specific teaching practices. The format of teaching is mainly the traditional one. The lecturer stands at the board and does most of the “telling” with rare interaction with students. Interestingly, this rare interaction is an expression of lecturer’s affective sensitivity (SSA) to large cohorts of students:

> “In an audience of 200 students, if you discuss with 2 – 3 of them, these probably will be the strongest students and the other will feel bad. … And finally nothing will remain on

the board… Here, we talk about masses of students and how to achieve a practical result for them. That's the point!"

In elementary or in secondary classrooms, interaction is a key part of current visions of effective mathematics teaching (Stein, 2008). But how realistic could be to expect interaction in a university amphitheatre stuffed with 200 students? The lecturer here cares about students' who "will feel bad" if he interacts with 2-3 of their colleagues. At the same time he points to the effectiveness of teaching for 'masses' of students as practically opposite to interaction probably due to the time the last requires. His perspective could contribute to a discussion of what sensitivity to large cohorts of students could mean and thus could be of help to reassess this element of the Triad.

The analysis of a teaching episode that is typical of this lecturer's teaching follows (Table 1). In this episode, the proposition "if a series converges then the sequence of the series is a null sequence" is taught. The concept of series and the definition of a convergent series had been introduced before. Also, the harmonic series had been given as an example of a non convergent series, still written on the board.

Episode	Teaching practices
[1] L: Now, there is a basic question: Given the sequence a_k, we want to see if we can add them i.e. *[he writes]* if the series of a_k, converges or not.…	Posing a problem (MC)
[2] I shall show a proposition…. They have given me a sequence a_k, ok? If the series of a_k converges, then necessarily the sequence a_k must tend to 0*[he writes]*…	Formulating the proposition (ML)
[3] Of course, because you are advanced now, you will ask if the inverse holds. If the inverse held, just taking a look at a_k and seeing that it tends to 0… would be enough. I would say that the series converges. If it didn't tend to 0, I would say that the series does not converge, and that would be all!	Connecting the proposition with the initial problem (SSC)
[4] However, I have already written an example for you [the harmonic series] ….This series tends to infinity.	Justifying (SSC)
[5] Attention here! This point [non – convergence of the harmonic series] remains up to the final assessment.	Highlighting (ML)
[6] The quick way [of proof] and I will show the slow way as well. Ok? ...	Evaluating (ML)
[7] I repeat again. We should not forget that a_k are the terms of the sequence. S_n is the sum of the first n terms of the sequence. If a series converges to $s \epsilon R$, then S_n also converges to s. *[he writes]*	Repeating (SSC)
[8] Now, I define a second sequence t_n as follows – I am going to write down for you the terms of this sequence. First, I set … let's say t_1, to be equal to 0. Then I set the 2^{nd} term of t_n to be equal to S_1, 3^{rd} to S_2… Ok? 4^{th} to S_3 etc. Namely, I set t_1 to be 0 - you can set everything you want. Let t_n to be S_{n-1}; t_n is S_{n-1} if $n \geq 2$. *[he writes]*	Explaining a process formally (ML)
[9] I want to define the sequence clearly. The books just write "consider the sequence S_{n-1}", but what is the S_{n-1} if n=1? Is it S_0? It is not defined. Ok?	Evaluating (ML)

[10] If you want to define this sequence S_{n-1}, you set a sequence t_n ; you set the first term and then you transfer the terms. Namely the 2nd term of S_{n-1} is the 1st term of S_n; the 3rd is the 2nd etc. Fine. … So, you crack the first term and then you get all the other terms of a sequence which tends to s.	Explaining a process informally (SSC)
[11] So the sequence you get tends to s. So t_n goes to s, too. So the difference of the two sequences goes to 0. *[he writes: "Then $t_n \rightarrow s$. So, $S_n - t_n \rightarrow s\text{-}s= 0$"]*	Inferring (ML)
[12] But what is the difference of the two sequences? For n≥2, the difference $S_n - t_n$ is the following: S_n is the sum of the first n terms of a_k and t_n is the sum of the first n-1 terms. Ok? Because $t_n = S_{n-1}$, so $t_n = \alpha_1+\alpha_2+\ldots+\alpha_{n-1}$. *[he writes]*. Thus the difference $S_n - t_n$ is… a_n, it is the only term left.	Explaining a process formally (ML)
[13] I have done all these analytically because the book writes "the difference $S_n - S_{n-1}$ is equal to α_n" and nothing more. This is how we calculate this difference!	Evaluating (ML)
[14] Now, if you don't like this way, you can prove the proposition using ε. Second proof – we will not set the sequence t_n. *[he continuous with the second proof] …*	Giving an alternative method (SSC)
[15] So, you prove a very useful proposition: you keep on hoping to add the terms a_k if they tend to 0. If a_k does not tend to 0 then you directly say "it is over".	Connecting with the initial problem (SSC)
[16] This proposition is very useful as a non - convergence criterion. Ok?	Evaluating (ML)
[17] For example: Someone gives you the sequence $a_k = \frac{k-1}{k+1}$ and asks you if the series of a_k converges or not. .. Then, he does not ask you anything!	Applying (ML)
[18] The first thing you have to do is to look at a_k . You say to him that a_k tends to 1 and not to 0, so the series doesn't converge. Ok? I.e. the first thing you look at is if the k-term inside the series tends to 0. *[he writes]*	Providing a solution method (ML)
[19] Therefore, the only interesting question about a series can be formulated in the case that the sequence of the series tends to 0. All the other series do not converge!	Refining the initial problem (ML)

Table 1: A teaching episode and its analysis (Translated from Greek)

In the above episode we see an example of how the lecturer attempts to help students to start their studies smoothly in practice. Students were used to more method-oriented teaching practices at school. Here, methods are also provided (e.g. in [18]) but in the context of a more global perspective: a problem is posed at the beginning [1] and it is refined on the basis of what has been proved at the end [19]. The same global perspective is identified in the other episodes, too. Also, technical processes are clarified and explicated [8]; what students need to know is repeated [7]; gaps found in textbooks are fulfilled [9], [13]; and all the explanations are written the board neatly arranged (e.g. in [1], [2] etc). This explaining of the mathematical content clearly and systematically seems to support students' learning. In fact, several students told the observer during the lectures' breaks that they consider this lecturer's way of explaining very "analytic" and the notes they keep from the board during the lectures very helpful for their studying. Further, the lecturer demonstrates aspects of advanced mathematical thinking [3], [4], [6], [11], [14], [15], [16], [17]. He uses verbal representations to describe a mathematical process [10] and the familiar to students natural language (e.g. in [15] “keep on hoping to add the terms”) to further clarify a process. He uses the pronouns “I” and “me” (e.g. in [2], [8]) giving to mathematics a human face. At the same time, he generates an air of relief (Pritchard, 2015) inside the amphitheatre (e.g. in [14] “if you don’t like this way, you can prove the proposition using…”) and retains an atmosphere of interpersonal conduct with each student using “you” (singular, e.g. in [17]). In his words:

> “It is as though I have a particular student in front of me … right here, and … you say to him ‘be careful! Here, I try to do this’ – but I do it for all students together.”

In terms of the TT, in this episode, we mainly see lecturer’s management of students’ learning (ML) in his teaching practices. However, this ML stems from Sensitivity to Students (SS). For example, the lecturer takes into account that some students cannot attend the lectures for personal reasons. Taking into account the broader macro context into which the students study is judged as an expression of his social sensitivity to students (SSS); based on his perception on what students may need to study he organizes the course with an e-class (ML) where all the students, even those who cannot attend the lectures, have access. Moreover, accommodating students’ possible difficulties with gaps found in the textbooks is judged as an expression of lecturer’s cognitive sensitivity (SSC) but it is instantiated for example by explaining analytically a new sequence [8] (ML) to bridge that gap. Also, taking into account students’ feelings in the case of interaction with others is judged as an expression of his affective sensitivity (SSA) even if it leads to a teaching closer to ‘showing and telling’ (ML). Thus SS is judged to be central for his practice. Possibly the large number of students that attend his lectures is also an impact of this sensitivity.

CONCLUSION - DISCUSSION

In this paper, we studied a lecturer’s Calculus teaching to a large cohort of first year mathematics students in a mathematics department. This lecturer is an exemplary case in terms of the large number of students who choose to attend his classes. His main goal was to help first year students to ‘start their university studies smoothly’ namely to overcome difficulties they might have in their move from school to university mathematics culture. He carried out this goal by supporting students’ learning of the advanced mathematical content and by introducing students to aspects of advanced mathematical thinking. In particular, teaching practices such as explaining; highlighting subtle aspects; repeating and providing steps and methods were intended to help students to overcome possible difficulties with the mathematical content and thus to support their learning. Teaching practices such as posing a problem and refining it; using alternative methods; relating mathematical ideas; representing and justifying were intended to introduce students into aspects of advanced mathematical thinking. Further, the organization of the course, mainly by using electronic sources (e-class), was intended to support students' learning, especially with regard to students who could not attend the lectures.

We interpreted the identified teaching practices in terms of TT's Sensitivity to Students and Management of students' Learning in an attempt to substantiate what could be meant by "taking students into account" in teaching large cohorts of first year mathematics students. This interpretation was simultaneously a process of reassessing TT's elements by identifying new possibilities and relations in the particular context. We found that, in a lecture context, Sensitivity to Students can be central in teaching practice and that Management of students' Learning, which is expected to be predominant in this context and is closer to 'showing and telling', can stem from this sensitivity. We also found that, at the particular context, the interaction between the lecturer and the students can be questioned by the lecturer's Affective Sensitivity to large groups of students and that Sensitivity to Students in the social setting of an amphitheatre can receive a social dimension. In the particular case of the lecturer we presented, this dimension of sensitivity seemed to create a positive learning atmosphere in the amphitheatre. Sensitivity to Students in the social domain has possibly a particular meaning in the context of large groups' university lecturing which may deserve a further exploration. In this study, lecturer's Sensitivity to Students was central in his teaching practice and affected the Management of students' Learning.

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