Students' work in mathematics and resources mediation at entry to university

Ghislaine Gueudet[1] and Birgit Pepin[2]

[1]CREAD, University of Brest, France, ghislaine.gueudet@espe-bretagne.fr,
[2]Technische Universiteit Eindhoven, The Netherlands

In this paper we study the use of resources by students in their mathematical work at the beginning of university. The institution offers a variety of resources: lecture notes, books, exercises, websites, to name but a few. Leaning on a theoretical framework by Rabardel, we argue that the university teachers expected an epistemic mediation of these resources, as they supported student learning of (higher level) mathematics. However, analysing two case studies (one in the UK and one in France) we observe that the actual use of resources by novice mathematics students corresponded to a pragmatic mediation, as they searched for worked examples and "reproduction techniques", all very similar to their use of resources at secondary school.

Keywords: epistemic mediation, pragmatic mediation, resources, secondary-tertiary transition

STUDENTS' WORK WITH RESOURCES AT THE SECONDARY-TERTIARY TRANSITION

The secondary-tertiary transition is a moment of important institutional evolutions: e.g. in terms of the mathematics taught; the mathematical practices expected from students; the institutional support for student learning (Gueudet 2008; Pepin 2014). In this article we claim that the resources available for students' work with mathematics, and the expected use of these resources (expected by the institution) significantly change from secondary school to university mathematics education. When entering university, it is assumed that students develop new ways to use resources, mathematical texts in particular. Previous research (e.g. Rezat 2010) contends that at secondary school even grade 12 students used their mathematics textbook mostly to search for worked examples, in order to learn rules and how to apply these rules to tasks similar to the tasks they worked on with their teachers in class. The same holds true at the beginning of university: Lithner (2003) shows that students' homework with textbooks was mostly oriented towards solving exercises; and that students searched for surface similarities between exercises, in order to choose a procedure. Our hypothesis is that this use of textbooks and other mathematical curriculum material by university students derives from similar practices at secondary school, in particular their use of textbooks. Hence, our research question is the following:

- which use of resources is expected by university mathematics teachers, and how does this compare with the actual use of resources by students?

In the next section we explain the theoretical frame we used, Rabardel's (2002/1995) instrumental approach. Subsequently, we present a case study from the UK TransMaths project[1], considering students' work in mathematics at a general level; and a case study in France concerning students' work in the area of Number Theory.

RESOURCE MEDIATION: THEORETICAL FRAME AND METHODS

In terms of theoretical frame we refer to the instrumental approach by Rabardel (2002/1995). He distinguishes between an artefact - produced by humans, for an aim of human activity; and an instrument - developed by a subject along his/her goal-directed activity with this artefact. The instrument incorporates the artefact (or parts of it) and a scheme of use for this artefact (Vergnaud 1998). Following Vygotsky (1978), Rabardel and Bourmaud (2003) consider that the goal-directed activity of a subject is mediated by instruments, in particular between the subject and the object of his/her activity. This mediation has a *pragmatic* value: the instrument permits to reach the aim of the activity; it contributes to the production of an outcome. At the same time it has an *epistemic* value: it contributes to the development of the subject him/herself and of his/her understanding of the object. This pragmatic/epistemic distinction has been stressed in particular by Artigue (2002) in her work about the use of CAS (Computer Algebra Systems). She identified pragmatic and epistemic values in the instrumented techniques developed by students: these techniques both permitted to reach an aim, for example solve an exercise; and to build new mathematical knowledge, enabling the students to solve further exercises. Drawing on the work of Rabardel, we developed the theoretical frame of the documentational approach (Gueudet, Pepin & Trouche 2012). In this approach, instead of artefacts we considered resources of different kinds, following the definition proposed by Adler (2000): everything that is likely to re-source the activity. Our previous research has mostly been concerned with teacher interaction with resources; our focus here is on students' use of resources. We distinguish between available/proposed resources and resources-in-use, comparing the resources available for students (resources offered by the institution), and the resources actually used. We already analysed the two cases presented here in terms of links between the Didactic Contract (Brousseau 1997) and the use of resources (Gueudet & Pepin 2015). In this paper we extend the epistemic/pragmatic distinction to the mediation realised by various kinds of resources (Gueudet, Pepin & Trouche 2012) intervening in students' work, and investigate these mediations. The resource mediation can be represented by the following figure (figure 1).
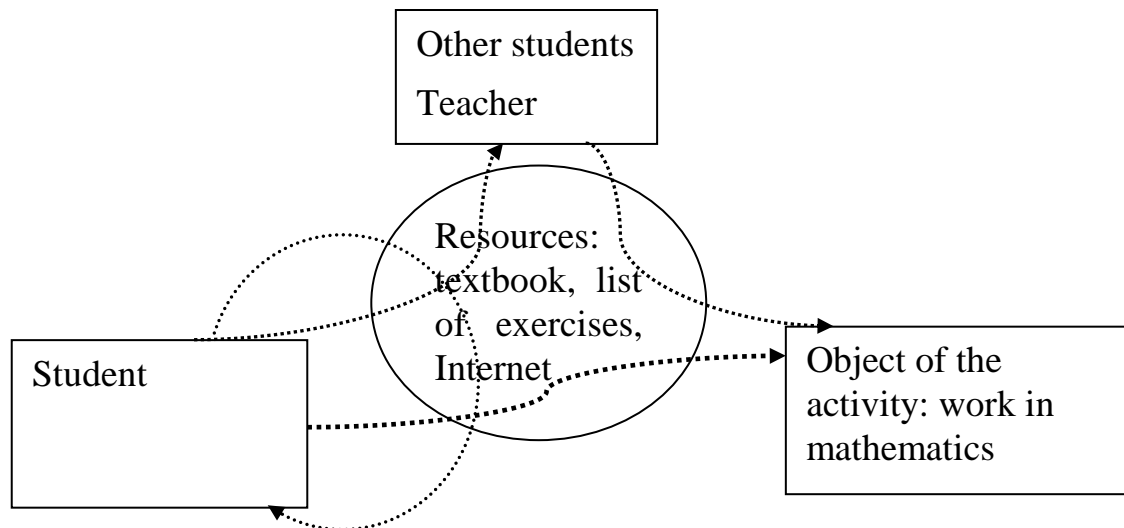
Figure 1. Resource mediations of students' work with mathematics.

Students at university use resources of various kinds: paper resources presenting the text of the lecture; their own notes; lists of exercises; textbooks; online resources found on the Internet; etc. These resources mediate the interaction between the student and the object of his/her activity; it has both a pragmatic and an epistemic value.

Both teachers' expectations and students' actual use of resources can be investigated through interviews with teachers and students, and we conducted such interviews in our two cases in the UK and in France.

In the UK, we explored the use of resources at a general level, at a medium-size university in a large city in the South of England. We mean by 'general level': for all mathematical content areas (taught during the first year), focusing on the general organisation of students' work. Beside conducting student interviews (with selected students), we surveyed all students of that particular mathematics course; we observed lectures, and interviewed lecturers, in addition to other support staff (e.g. teaching assistants at university) and teachers at secondary school. We also collected documents at both institutions (school, university, in the UK). For the work presented here, we selected a case study subject who is an ethnic minority student, Simar and his (ethnic minority) friends, all studying mathematics in the faculty of mathematics at the same university. We followed Simar over approximately two years: starting when he transited from a local upper secondary school into a university mathematics course, and into his second year at university. Over that period observations and interviews were conducted at several data points: (1) in his previous school (with his mathematics teacher/s); (2) at entry to university (with Simar and his friends; and with lecturers and support tutors); (3) towards the end of the first year at university (with Simar and his friends); and (4) during the first semester of his second year at university (as under (2)). For this study, interviews

with Simar (3); focus interviews with his friends (2); and interviews with selected lecturers (5) were analysed using our theoretical frame explained earlier.

The case in France has been conducted with a focus on a particular mathematical content, Number Theory. We collected data in a medium-size university where Number theory was taught in a first year teaching unit. During the academic year 2014-2015, we interviewed the teacher responsible of the course about the use of resources she expected from students. We also proposed an online questionnaire to the 140 students about their use of resources. We collected 85 answers (around 61 %). In 2015-2016 we complemented this study by proposing exercises as homework to a group of students, and interviewing three of them about their use of resources for solving these exercises. The collection of data at different levels, we claim, is likely to give us insights at different phenomena, which (it is hoped) complement each other.

CASE 1: STUDENT RESOURCES AND THEIR MEDIATION AT GENERAL LEVEL

From interviews with Simar (and his friends), who studied at City University, we could identify the main resources used in their first year of study: the lecture and lecture notes; the coursework; notes and tutorial notes made during tutorial and/or with his friends/study group. It was clear that these resources were quite different, in nature and quantity, from what students were used to at school: at school students had one textbook (which was portraying mathematics as something that one can learn by solving "tons of exercises"), whereas at university they were expected to work with many resources: e.g. different textbooks; lecture notes; examination papers; etc.

At City University the main resources were clearly the lecture, usually in halls of up to 300 students, and the lecture notes provided by the lecturer/professor (sometimes supported by a textbook). Different lecturers had different styles of lecture notes. Some lecturers would produce hand-written notes projected onto a screen (and talk students through the content during the lecture) (e.g. calculus). Simar and his friends would copy these notes, most of the time with little or no understanding.

However, Simar talked quite enthusiastically about one of his lecturers, and he pointed to what he (and his peers) would regard as a good lecture and lecture notes.

S: Geometry: the feedback we got from geometry is, basically he's faultless. He's brilliant, he's excellent; the lecture's engaging, the notes are available- clear notes. You can use the notes for the coursework.

Int: The notes are handwritten?

S: Yeah handwritten notes yeah. And they actually, you can see the kind of proofs- he doesn't give too much away, but it's just enough to get you thinking in the coursework's, which is excellent. ... Because like, what students are finding is that they can go, because the lectures, they're not gonna walk around with the lecture twenty-four, seven are they? They need something to take away from the lecture and you know, they're gonna ready at home, they're gonna read it, and they understand it. ... And they can go to the tutorial, ask

whatever questions and do the questions with confidence, knowing that they've done well like because everything's there, available. They don't need to go anywhere else, and if they do, the tutorial's available or the office hours. So really it's probably one of the best.

Int: So do you think they understand because in the lecture he explains well, or do you think they understand because the, it's so well-prepared and written out?

S: I think mainly it's mostly well-prepared, definitely, and then to accompany that, the lectures are brilliant as well. Yeah it's really, really kind of funny. He catches your interest..." (DP5, Simar)

Another lecturer would provide notes that students had to read in advance of the lecture. These notes had "holes" that needed to be filled in. During the lecture the lecturer would then discuss the content, and subsequently fill in the "holes". In such a way, students were not only obliged to prepare the lecture in advance (in order to be able to understand the notes in the lecture and fill in the missing text), but they also needed to attend the lectures to have "complete" notes.

In addition to lectures, the coursework (provided once a week) was to support student understanding of the lecture, through exercises. Simar and his friends/learning group were clear that unless the coursework was well aligned with the lectures, it did not help their understanding of the subject area (see Calculus as compared to geometry lectures/coursework). Indeed, in some cases students did not know what to ask in tutorial time, or in lectures, so little had they understood of the topic area. Other resources included textbooks (suggested/approved by the lecturers), but these were seen as less helpful than the lecture notes and coursework (provided by lecturers and tutors), in particular as students were often "learning to the test". However, the same resources (e.g. lecture notes) were often evaluated very differently by students, in terms of support for their learning, so much so that Simar (as student representative) had asked for a change in form and practice concerning lecture notes: as students did not want to be presented with "one slide after another".

Interestingly, in terms of lecture notes students distinguished between different types of notes: (1) "understanding notes" were well prepared and developed, apparently useful for understanding and coursework (and tests); (2) "comfort notes" were those that students did not understand but "you've got to have the notes" and "you have gone to the lecture", which in their views helped for knowing what to study for revision and examination purposes; and (3) "motivation notes" were provided on the student web, before the lecture, and which apparently "makes you want to come to the lecture … because they are different" (DP5, p.4).

At the same time institutional practices, such as lectures, and accompanying resources played a crucial role in the ways that mathematics, and what it meant to "do mathematics" was portrayed. On the basis of video footage of selected lectures and pre- and post-video stimulated recall discussions with lecturers one could identify meanings that were attached to particular practices. Particular lectures reflected the kinds of things that a "rigorous mathematician" may need to learn:

- 'reasoning and proof' based thinking and practices were expected to be developed through Geometry and Linear Algebra;

- 'procedural fluency' (methods) was seen to be developed through Calculus;

- practical and context relatedness was regarded to be developed through Statistics.

However, it was clear that during lectures student would not learn how to work as a "rigorous mathematician", neither did students expect this from lectures and lecture notes. What students wanted were "help notes" for doing their course work, and worked examples suitable for studying for examinations.

"The only way I understand to do my work is, when I'm doing my coursework and there are help questions to do your coursework, and this is how I tend to them more and during the tutorials, and I think the tutorials and the courseworks are more helpful than, the lecture. The lecture you just get the notes." (DP5 Focus group interview)

In terms of mediation of resources, in particular lecture notes, it can be argued that for students they had mainly pragmatic value: Simar and his friends were content, if they were given the "instruments" to do their coursework and examination questions. However, for university lecturers the mediation of (for example) lecture notes had epistemic value: they wanted to develop students into "rigorous mathematicians", and lecture notes (and lectures) would show them how 'rigorous mathematicians" worked. How that could be learnt was not clear, except for alignment with what the lecture notes showed as examples. In fact, at City University one lecturer realized the problematic, and he had started a module on "writing mathematics" which was to provide students with the language they needed to appreciate the epistemic side of the subject.

CASE 2: STUDENT RESOURCES AND THEIR MEDIATION IN NUMBER THEORY

Our investigations took place in a first year teaching of Number Theory spanned over twelve weeks, with four hours each week (two hours of lecture, and two hours of tutorial). The first half of the course concerned logic, sets and combinatorics; the second half more directly number theory, with Euclidean division, Euclid algorithm, prime numbers and congruencies.

Use of resources by students and pragmatic mediation

In 2014-2015, at the beginning of this course the students were provided with a "polycopie", which included more or less the text of the lecture; and a list of exercises. They could also access complementary resources, on the webpage of the teacher responsible for the teaching: previous exam texts; references of books; links towards online exercises. We proposed an online questionnaire to the 140 students concerned and obtained 85 answers. Only 52% of these 85 students declared that they found the polycopie useful. They considered that the text of the lecture of their

teacher was enough, and used the polycopie only before the final exam (83%). Moreover 90% would have liked to find worked examples in the polycopie; and 44% looked for additional resources on the Internet, in particular worked examples. We contend that these answers evidence that the polycopie mediation remains pragmatic for the students, similar to their use of textbooks at secondary school where number theory is limited to the application of some techniques (Battie 2010). Alike students in the UK, they search for worked examples in order to reproduce techniques, whereas the teacher expects that the polycopie has a strong epistemic value, and is used to work on the course: learn definitions, understand proofs of theorems etc. (declared by the teacher responsible for this teaching).

Mathematical resources for Number Theory

In 2015-2016, following the results of the questionnaire evoked above (a report about the answers was presented to the teachers of the number theory unit), no polycopie was given to the students. A book was recommended instead, together with a website, Braise[2] proposing exercises associated with different mathematical texts: description of methods, extracts of the course, hints, partial solution etc.

The teacher proposed to the students the following exercise as homework:

Exercice 28

Soit n un entier positif, prouver que $2n + 3$ et $n^2 + 3n + 2$ sont premiers entre eux.

Figure 2. Exercise given as homework (Let *n* be a positive integer, prove that $2n+3$ and n^2+3n+2 are coprime).

This exercise can be solved by searching for *a* and *b* such that *a(2n+3)+b(n²+3n+2)=1*. There is a specific difficulty in this exercise, since *a* had to be itself of the form *un+v*, where *u* and *v* were constant integers.

A student can solve this exercise without any explicit use of resources; nevertheless, since the homework was given at a stage where such tasks are not yet familiar to students, we consider that this will probably not happen. Amongst the resources provided by the institution, the students can use their course notes, in particular to find the definition of coprime. They can also use the notes taken during the tutorial, where an exercise using a similar method has been done: *"Let m and n be integers, such that m divides both 8n+7 and 6n+5. Show that m=±1"(Exercise 8).* In this exercise, the students also need to find a linear combination of *8n+7* and *6n+5*, which does not depend on *n*. The students can also visit the Braise website; they could find on it a method entitled "Determine if two integers are coprime" which can also be used here as a resource. It does not mean that the students will only need to reproduce the same method: in particular, the presence of *n²* in exercise *28* requires a

significant adaptation of the method. We claim that, in such a case, the resources mediation has an important epistemic value.

16 students did the proposed homework; ten of them proposed a correct solution, and for six of them proposed a wrong solution. We met three of these students for an individual interview about the resources they used to solve the exercise: Brian, and Franck, who did not succeed; and Tom who found a correct solution. Tom used exercise 8 that he found in his tutorial notes, and correctly adapted the method. Franck used exercises that he found on the Internet (but not on BRAISE), which he identified as useful in terms of including the idea of Euclidean division of polynomials, and divides n^2+3n+2 by $2n+3$.This lead him to conclude that the gcd is (-1/4), "so is ± 1 up to a constant multiplier"). Brian searched his lecture notes, found the property "if p is prime and n an integer, then p and n are coprime". He tried to apply it but realised that $2n+3$ is not always prime. Then he searched grade 12 textbooks, found the linear combination method but only with constant coefficients, and thought that the coefficient cannot depend on n. We claim that, while for Tom and Franck epistemic mediations of the resources took place, for Brian the mediation was limited to a pragmatic aspect. He searched for a method that he wanted to apply without any adaptation. Franck took personal initiative searching for resources that where not proposed by the teacher. He tried to build an original method, but was not successful in controlling its correctness. It is noticeable that, while Brian came directly from secondary school to university, Tom and Franck have had previous experiences at university: Tom did a first year of law studies before deciding to study mathematics, while Franck studied two years of "computer science and networks". We can assume that the influence of secondary school was less important for them, as they had previous experiences at university.

CONCLUSIONS

The changes in resources (for teachers and for students), and in the use of these resources, at the secondary-tertiary transition have been under-researched. We contend that the study of "resource use" is an important theme for research, likely to deepen our understandings of teaching and learning processes (initiated or supported by resources) at the beginning of university. In the two case studies presented here we observe that the institution provides the students with numerous resources, mainly mathematical texts. According to the teachers, the epistemic mediation of these resources should support students transiting from school to university mathematics, both in their ways of learning mathematics (more self-regulated work, more autonomous reading of mathematical texts, see e.g. Farah 2015), and in their ways of "doing" mathematical work, so that it would become similar to the work of a "real mathematician". However, asking the students about their actual use of resources led to a different picture. The learning at university seemed to be based on listening to the teacher (in lecture), writing down notes, trying similar worked examples, reading the polycopie for exam preparation (in France) – in other words an

alignment based on a kind of apprenticeship learning. We contend that, based on the two cases we studied, the pragmatic mediation of resources took over the epistemic aspect, at least in the first year. Students used worked examples and lecture notes in order to produce the desired results. At the level of mathematical content, in our case number theory, we detected a potential epistemic aspect in the use of worked examples and lecture notes, when a significant adaptation of a given method was needed. From our study we claim that the "enculturation" and "alignment" processes associated with the change from school to university mathematics education take longer than expected (by university staff), and more awareness and didactical flexibility (from the side of university staff) might help students to bridge this gap more successfully.

NOTES

1. TransMaths project, University of Manchester: http://www.transmaths.org

2. http://braise.univ-rennes1.fr/braise.cgi

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