Engineering students' use of visualizations to communicate about representations and applications in a technological environment

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Research about learning styles show that many engineering students are visually inclined. Therefore, we study engineering students' use of visualizations to communicate while solving mathematical problems. Based on a framework for mathematical representations, visualizations and mediation, we set up an explorative study with the visualization tool Sim2Bil, which combines a simulation of two cars, velocity graphs, and an input for velocity functions. We asked three engineering students to create a simulation for the cars with certain conditions and studied their visualizations related to verbal, graphical and symbolical representations. The visualizations were Sim2Bil-based, paper-based and gestures and these were all used in different stages of the problem solving.

Keywords: digital visualization tool, engineering education, kinematic simulation, representation, visualization.

INTRODUCTION

Our research is situated within mathematics education in engineering studies. Research has shown that many engineering students have a tendency to perceive and process information visually, for example in forms of pictures, diagrams and flow charts (Felder & Brent, 2005; Hames & Baker, 2015). Therefore, it is important to study aspects of visualizations in mathematics education for engineers.

Technology has offered new ways for visualizations in mathematics education by, for example, allowing users to construct and drag geometrical shapes and graphs of functions. As such, technology changes mathematical practices (Artigue, 2002; Hegedus & Moreno-Armella, 2014). Also, future developments in education may lay in expanding technology-supported collaborative work between students (Lowyck, 2014). In this sense it is important to study visualizations in how engineering students communicate with each other during problem solving.

The focus of the study is on how engineering students use visualizations in their mathematical communication. We carried out an explorative study on students working on a mathematical task within a kinematical context and with the digital visualization tool *Sim2Bil* (described in 'Methods'). Especially, we focus on how they socially create and communicate meaningful visualizations.

Within engineering education an important mathematical topic is calculus, in which students learn about concepts such as derivatives and integrals. These concepts are useful for modelling certain phenomena in an engineer's practice. In our study we focus on *integrals*. Previous studies conclude that technology can support students in their graphical approach to integrals (Berry & Nyman, 2003; Swidan & Yerushalmy, 2014). In these studies, the tasks they used included graphical representations and the integrals were not connected to contexts. To supplement these studies, we opted for using a kinematical context for integrals, because kinematical concepts such as *velocity* and *acceleration* are basic phenomena in engineering (and in physics). Also, digital technology enables objects to move on screen. Therefore, we wanted students to use mathematical concepts for speeding up or slowing down objects. So, students were to use integrals for the simulation of movement within a technological environment.

The aim of our study is to investigate how engineering students use visualizations in their communication about representations and applications within a kinematical simulation context.

THEORETICAL FRAME

In our study students' interactions with mathematics, Sim2Bil and each other are analyzed through a socio-cultural perspective. We will explain key terms within our study: visualizations, representations and mediation.

Visualization has been used differently in the literature. Presmeg (2006) describes visualization *"to include processes of constructing and transforming both visual mental imagery and all of the inscriptions of a spatial nature that may be implicated in doing mathematics"* (p. 206-207). According to her, when someone constructs a visual arrangement (e.g. draws a graph of a function), there is a visual image in the mind guiding this creation (p. 206). The socio-cultural approach taken in this paper offers a different perspective. Visualizations organize and structure information in a spatial way, and thereby offer a means of communication. Visualizations may be mental, but they also are a social, complex system for communication.

The term *representation* is often defined by using the word represent in itself. Janvier (1987) introduced four representations for functions: 1. situations, verbal description, 2. tables, 3. graphs, and 4. formulae. Zandieh (1997) adapted the naming of representations to the derivative: 1. graphical (slope), 2. verbal (rate of change), 3. physical (velocity, acceleration) and 4. symbolical (difference quotient). Using and adapting these terms to the concept of integral, we can say that an integral has different representations. First, an integral can be represented verbally, for example, by stating "an integral can be calculated by an anti-derivative" or "an integral is a limit of a summation". Second, an integral can be represented graphically as area under a graph. Third, an integral can be represented symbolically: $F(x)=\int_0^x f(t)\,dt$. These mathematical representations (verbal, graphical, and symbolical) have been developed by mankind through a long cultural history. They are cultural artefacts.

When set within a kinematical context, an integral can have special interpretations. To capture the kinematical context, Zandieh (1997) introduced the term 'physical representation'. We will adapt this by speaking of kinematical applications. Kinematics is a sub-discipline of physics, in which motion is studied through concepts, such as *velocity* and *acceleration*. Within a kinematical context, if one has a function $v(t)$ for the velocity of an object, then an integral models the distance travelled $s(t)$. This kinematical application of an integral is not mutually exclusive to the three mathematical representations. Instead, stating that "the integral of the velocity yields the distance travelled" is a verbal representation, which could also be expressed graphically or symbolically. Engineering students will need to learn about such representations and applications, and about which one is needed when. Hong and Thomas (2015) point out that students need to develop considerable flexibility in this, in order to do well in their studies.

Finally, we want to explain the term *mediation*. In our interpretation of the social world around us, cultural tools play a role in mediation. Tools are important resources and function as media, through which we communicate knowledge in the social context (Vygotsky, 1978). Cultural tools within this study are, for example, Sim2Bil, gestures, language and mathematical representations. Tools have a dual nature: material and ideal. Tools are material in that they are embodied in artifacts. Language is material in the configuration of writing, sound waves or as neuronal activity (Cole, 1993). Tools are ideal in that they contain "in coded form the interactions of which they were previously a part and which they mediate in the present (e.g. the structure of a pencil carries within it the history of certain forms of writing)" (op.cit., p. 249).

In the technological environment we created, the students can use mediating tools in different ways for their communication, and we will focus on visualizations. Our research question is: *how do engineering students use visualizations to communicate about representations and applications in a technological environment?*

METHODS

The methodological approach seeks to document students' communication using Sim2Bil. The object of study is the interaction between students. We set up a small-scale controlled environment, which was organized outside the normal lectures. The students volunteered to take part in our experiment.

Participants

The participants were three engineering students from our university and indicated by pseudonyms: Sam, Erik and Tom, 20-25 year old. The students had almost finished their first year, which included courses in calculus, linear algebra and physics (including kinematics). Sim2Bil was not familiar to them.

Sim2Bil

Figure 1 shows the digital visualization tool Sim2Bil. There are four windows. In the top left is a window where two cars can drive from a starting line to a finish line.

One car is green, one car is red. This is the *simulation window*. In the bottom left is a window with two graphs. One graph is for the velocity-time function of the green car and the other graph is for the red car. This is the *graph window*. In the bottom right are the velocity functions for each of the cars. Here, parameters for a 3rd degree polynomial can be inserted to create a velocity function. This is called the *formula window*. The fourth window includes a menu, which is unused in the study.

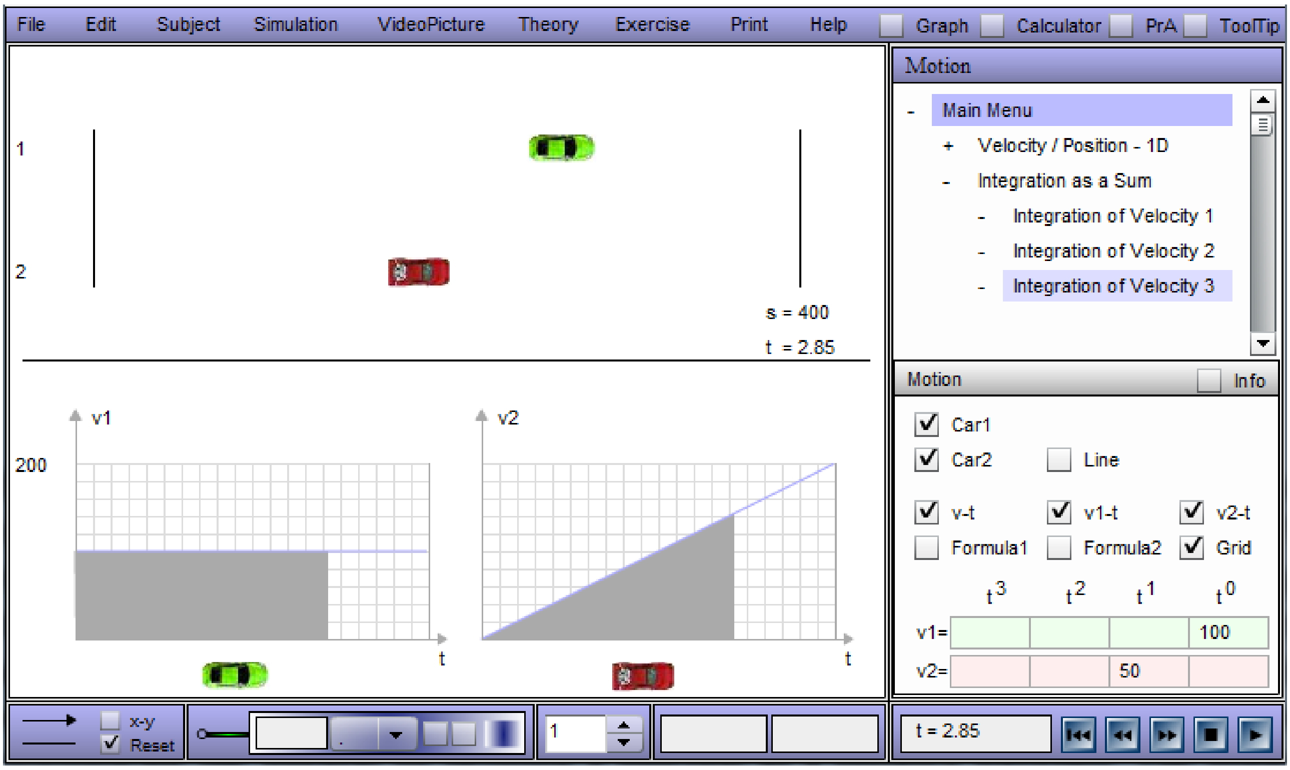

Figure 1: The interface of Sim2Bil for a kinematic simulation

A user of the tool can fill parameters for the velocity functions for each of the cars. By pressing the *Start*-button (bottom right corner), the cars will start running from the starting line to the finish line. The distance between start and finish is set to be 400 meters. The areas under the graphs will be shaded grey in an animation, whereby the shaded areas increase while the time runs (it is the same scale for both graphs). These areas represent the distance travelled, but this is not communicated explicitly.

By default, there were already two functions given in the formula window:

v1=100 (for the green car)

v2=50t (for the red car)

With these given functions a user can start the simulation by pressing the *start*-button. A user then sees how the cars run differently in the simulation window and at the same time sees two different graphs in the graph window. This is an easy introduction to the functionalities of the tool. To summarize: in the tool the symbolical representation generates simultaneously the graphical representation and the kinematical simulation.

The task

The students were asked to press the *Start*-button to see the cars run. The default setting made the cars run with different speeds, but they finished at the same time. This 'finishing together' constitutes a certain condition for the cars' trajectories. The students were asked to fulfil a similar condition in following task:

Find the velocities of the green car and red car (v1 and v2 respectively) so that v2 is half of v1 when they reach the finish line simultaneously at 4 sec. Can you prove that your answer is correct?

The task was offered on paper in a verbal representation. Symbolically, the conditions for the simulation of the cars may have been written as:

At t_{finish} you should have $s1=s2=400$ and $v1=2*v2$.

The task is open to different approaches, and therefore we expected it to generate collaboration between students. A trivial approach is to reverse the velocity functions of the cars. If the students suggested this, they would be asked to find other ways. It was anticipated that students would see relations between the distance travelled by the cars (in the simulation window), the shaded areas under the graphs (in the graph window), and the integral of the velocity functions (in the formula window). The representations can have different roles. The *simulation window* shows animated distance and velocity, and offers validation whether both cars get to the finish line at the same time. The *graph window* can, amongst others, offer information on the velocity at the finish line. The *formula window* asks for performing symbolic manipulations up to 3^{rd} degree functions, which constrains possible solutions.

The tool was set up on a laptop in front of the students. Additionally, students had availability over paper, a pen and their own handheld calculator.

Data collection techniques

To capture the students' interaction with Sim2Bil and how they communicated with each other using verbal expressions, visualizations and gestures, the students' work was video recorded with two cameras. One camera was directed at the students and their writings. The second camera was directed at the computer screen to capture mouse movements and students' input within the interface. The first author of this article was present, introduced the interface and task to the students. Students' notes were collected at the end. The students were not given a specific time for their work and it turned out that they spent 26 minutes on this task.

Data analysis

For the data analysis multimodal transcriptions (including written and spoken words, other writings, and gestures) were applied to the video recordings. The transcripts were coded and put into categories. We studied the instances, in which the students used visualizations in their work. The visualizations will be related to the different representations they used.

FINDINGS

The three students Sam, Erik and Tom were sitting from left to right. Erik pressed the *Start*-button in Sim2Bil and the three students watched the two cars starting together, running at different velocities, but finishing together again.

The students started to discuss the task. To meet the conditions in the task, Erik started explaining the cars' paths by making a gesture of how the cars would run. He used one hand for each of the cars, as if he was replaying the simulation window. Figure 2 shows this gesturing.

Figure 2: Excerpt of Erik's sequence of gestures of the cars' movement

He explained verbally the cars' paths:

5	Erik:	The red car should, the red car should speed up and then it should [his hands simulate the cars] slow down and then should this, the green car catch up. And then when they reach the finish line it should be half of the speed as the green car is when they drive into the finish line.

So, at the start Sim2Bil mediated a kinematic situation by showing the cars running in the simulation window. This kinematic situation was mimicked by Erik. Gestures and a verbal description of the kinematics were used by him and mediated to his peers how the cars' would need to run.

While Erik and Tom started a discussion on the conditions, Sam started to calculate on his own. After some minutes Sam wanted to verify his calculations. He asked Erik to insert parameters into the formula window in Sim2Bil:

58	Sam:	Write one half
	[Erik writes in a wrong number, and Sam corrects him]	
67	Erik:	One half. There I got one half
68	Sam:	No, then it's wrong [looking at the graphs at the screen]
69	Erik:	I tried to take minus fifty there [pointing at the table on screen] and then it will go like that. So, they will reach at the same time, but it will stop there. It shall be half the speed when it stops there. So, it must have a [making a gesture of a parabola, see Figure 3]

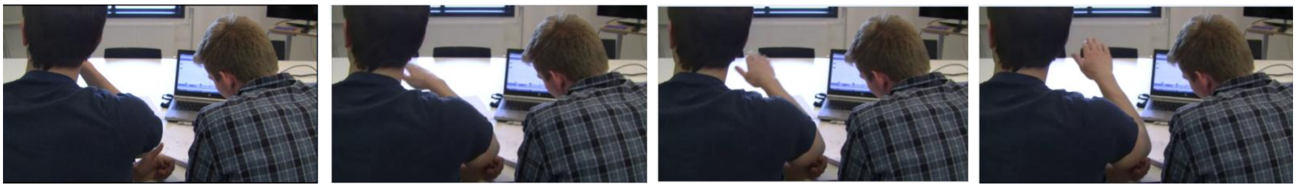

Figure 3: Excerpt of Erik's sequence of a gesture of the parabola

In this episode, Sam came up with velocity functions (which were incorrect) and he suggested using Sim2Bil for verifying. They observed the graphs of his functions within the graph window, and this alone served as verification. They didn't need to see the cars running in the simulation window, but merely a visualization of the functions through its graphical representation. Thus, the graphical representation in Sim2Bil mediated whether Sam's functions met the conditions.

In 69 Erik gestures a parabola, which is another visualization of a graphical representation. So, Erik's parabolic gesture mediated a new idea.

After discussing more, Erik suggested to simplify the problem by letting one of the cars run with constant speed. They let the green car have the same velocity as it had at the beginning of the work (v1=100). Tom drew a sketch of two graphs (one for each of the cars), for which the enclosed areas needed to be equal to fulfil the conditions in the task. Figure 4 shows Tom's sketch, which is a visualization that mediates his thoughts to his peers. Here is what they said and drew:

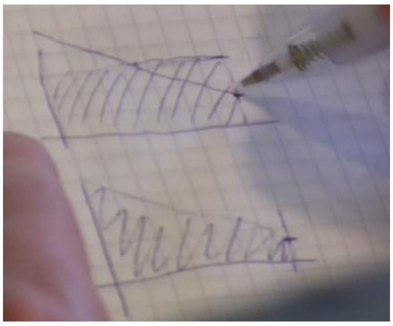

Figure 4: Tom's visualization of graphs

148	Tom:	And then you have the other one [draws the bottom graph in Figure 4]. That should just, that should end on half of here right [draws a point to the right of each graphs]? So, it should hit here. And then it should begin up here [draws an oblique line], and it shall have the same area [shades area under bottom graph]
149	Sam:	Like the other one
150	Tom:	Like the other one. Ehm, so then I mean that you can just draw a line from the middle there then. Up like this I was about to say. So the area here, so you can find out how high up it must start. And the slope on this one here, because it should, because… yes. You only need the same area on this one if you think trigonometrically then
156	Sam:	But what it is about is to find one…the area and a function for the y [the oblique line]

Figure 4 and 5 show the geometrical approach. Erik started from a constant velocity for one car, and took the distance driven as area under the graph (a graphical representation of integrals). This was a rectangle. Thereafter, they cut a triangle off to make a trapezium (a rectangle + a triangle) for the second car. So, Tom's first draft helped to develop and communicate the approach of using area calculation for rectangles and a triangle (they did not explicitly say that they wanted to avoid anti-derivatives). Then Sam filled in details to make a detailed graphical representation, see Figure 5. From there they created a velocity function for the second car.

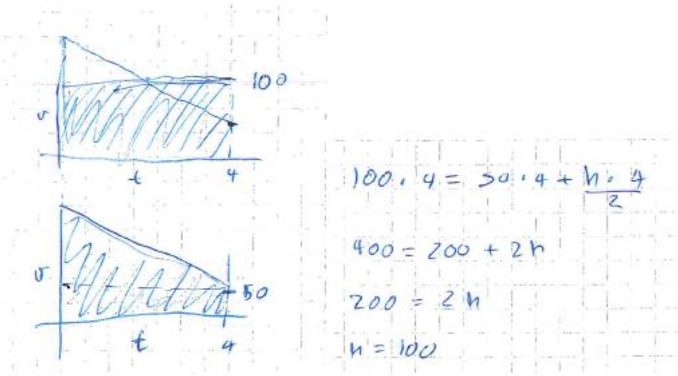

Figure 5: Calculating areas

The students ended with a graphical representation of two functions and the areas under their graphs. With the vertical *v*-axis, horizontal *t*-axis and proportionally measured values, this has the characteristics of a culturally mediating artefact. This picture could have been mediated by Sim2Bil, because the students could have copied the graph from the tool. Also, it could have been mediated in the courses that they had followed. To reach this stage, they first needed a visualization in the shape of a sketch, then a detailed drawing before they could reach a symbolical representation. Thereafter, they could fill in the parameters into the formula window of Sim2Bil to verify their symbolical representations. They were cheering for the cars when these finished exactly together at the finish line.

CONCLUSION AND DISCUSSION

We have studied engineering students' use of visualizations to communicate about representations and applications while using Sim2Bil. We observed three different media for visualizations:

- Sim2Bil-based visualizations: the cars running in the simulation window and the graphs of the functions in the graph window
- paper-based visualizations: sketches and detailed drawings
- gestures: mimicking the kinematic application (showing how the cars would run), and indicating a parabolic graph.

Although the task could have existed without Sim2Bil (by symbolical calculations), the tool was central in the task by offering visualizations. At the start, it served to introduce the students to the task, showing two cars run different trajectories, and finishing at the same time. Midway and at the end, the students used the tool for discussing and verifying, whereby both the graph window and the simulation window were used. So at the start, the tool mediated the mechanism for the kinematic simulation, while later the tool mediated the verification by giving feedback on the inserted parameters. Also, the mediating tool could be observed to be material and ideal. For example, the *Start*-button is a material area on which to click (and no real button) and it is ideal as a familiar sign developed through a cultural history.

The paper-based visualizations were used by the engineering students with the aim to produce symbolical representations for the formula window. For this, they needed to make sketches for organizing, discussing, and elaborating their ideas. So, the paper-based visualizations were tools in the mediation to find symbolical representations.

Finally, the gestures were mainly used to mediate ideas at the beginning of the session, when the students were exploring and explaining to each other the conditions of the task. Table 1 summarizes how the students' used visualizations in their communication about representations and applications of the integral. One cell remains empty: the students did not use gestures to produce symbolical representations.

		Mathematical representations			Kinematical applications
		Verbal	Graphical	Symbolical	
Visualizations	Gestures	explore explain	explain		explore explain
	Paper-based	discuss organize	elaborate	produce	discuss, organize elaborate
	Sim2Bil-based	discuss	verify	enter	introduce discuss, verify

Table 1: Use of visualizations in communicating about representations and applications

Of course, the results in this study are affected by the functionalities of Sim2Bil. If another digital tool had been offered, and a different task had been asked, the visualizations used by the students would have been different. Nevertheless, we see a variety in visualizations used by the students. This tells us that an environment such as Sim2Bil offers rich opportunities to visualize mathematics (e.g. the cars moving), and to connect different mathematical representations and applications. This sheds light on how we could enrich technology-based problem solving with visual aspects, not only for engineering students, but also for other students in higher education.

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