Teaching Calculus in engineering courses. Different backgrounds, different *personal relationships*?

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In this paper we seek to further investigate whether having different backgrounds influences Engineering teachers' views of Calculus and shapes their opinion of how the subject should be taught, and whether these views affect their actual teaching practices. Our research is based on an institutional perspective and employs Chevallard's Anthropological Theory of the Didactic (ATD), in particular the notion of personal relationship, and we analyse the possible impacts of institutional choices on an individual's practices. Our data seem to indicate that even when they occupy the same position in the same institution, teachers with different academic backgrounds hold quite different personal relationships with the contents of their Calculus course, and that this has a significant impact on their practices.

Keywords: calculus, engineering, university teacher education, personal relationship, Anthropological Theory of the Didactic (ATD).

INTRODUCTION

In many science and technology programs, Calculus is among the first courses taught. It is considered one of the most important early courses in engineering, allowing students to subsequently study and model real problems in ways that can be applied to their professional lives. Despite this, Calculus instructors often emphasise the application of techniques, the memorisation of definitions and the manipulation of formulae, rather than the acquisition of notions that are directly relevant to the practice of engineering. This can result in students failing Calculus and abandoning their professional ambitions (Ellis, Kelton, & Rasmussen, 2014). Regarding this issue, Christensen (2008, p.131) has pointed out that "it can be quite difficult to connect the abstract formalism of mathematics with the necessary applicable skills in a given profession", and that this could create a "gap in the students' ability to use mathematics in their engineering practices".

In general, Engineering courses are organised into two main groups: general science courses such as mathematics, chemistry and physics; and technical courses, which are specific to each branch of Engineering. Under this system, students in their first years of study may be unable to see where and when they will practically apply the mathematics and physics they are learning. In addition, they may find it challenging to recognise and apply this knowledge in later courses. As Harris, Black, Hernandez-Martinez, Pepin, Williams, & TransMaths (2014, p.334) conclude, "mathematics should be embedded with the engineering principles being taught. There [is] a danger that when mathematics becomes isolated from its use in engineering, the opportunity to foster a perception of its use-value in the wider sense [is] lost." Research on the teaching and learning of Calculus and analyses of students' difficulties have spurred growing interest in teachers' practices (Rasmussen, Marrongelle & Borba, 2014), which opens a new avenue of research in postsecondary mathematics education.

As part of this trend, Pinto (2013) recently analysed two lessons on infinitesimals given by two different teaching assistants – each with a different level of experience – using the same lesson plan. The analyses show that their different beliefs, objectives and levels of confidence in various resources resulted in two substantially different lessons. For the author, "a more specific contribution of this study refers to the ways in which teachers assistants' pedagogical content knowledge, or lack of it, affected the lessons" (p. 2424). This suggests that the teaching practices of university instructors are highly influenced by their own experience. Regarding this issue, in Hernandes Gomes & González-Martín (2015a), we studied the vision of mathematics held by engineering mathematics teachers with different academic backgrounds to understand how these backgrounds shaped their teaching practices. Our data revealed differences in the way these teachers approach topics such as mathematical rigor and approximation. Using tools drawn from the Anthropological Theory of the Didactic (ATD), and specifically the notion of *personal relationship*, we analysed data from interviews with engineering students studying under those teachers (Hernandes-Gomes & González-Martín, 2015b). Our results seem to indicate that elements of the teachers' *personal relationship* with mathematics emerge in the students' interviews, in particular those elements pertaining to rigor and estimations. These preliminary results motivate our current research agenda. We seek to further investigate whether having different backgrounds influences Engineering teachers' views of Calculus and shapes their opinion of how the subject should be taught, and whether these views affect their actual teaching practices.

THEORETICAL FRAMEWORK

Some teachers receive their initial training in one faculty but eventually teach in another, while others bring their professional experience into the classroom. We wish to determine whether these different backgrounds influence teaching practices and the way instructors prepare courses, and believe an institutional approach is appropriate for this purpose. We therefore applied tools from Chevallard's (1999) ATD. According to ATD, an *institution* I (in a broad sense) is a social organisation which allows, and also imposes on its *subjects*, ways of doing and thinking proper to I (Chevallard, 2003, p.82). Human activity can be modelled in terms of *praxeologies*, which are defined by the types of tasks carried out, the techniques that allow tasks to be completed, a discourse to justify the techniques used, and a theory that explains and justifies the discourse. The type of tasks and techniques allowed or promoted by an *institution* – together with the discourses that justify these techniques – have an impact on the individuals who belong to the institution.

A *subject* is defined as every person x who occupies any of the possible positions p offered by I. For our purposes, we may use the example of a faculty of engineering (I_1) which offers several positions, including teacher (in various departments) and student. The types of tasks, as well as the techniques and discourses available, are different for these two positions. It is possible for the same individual to have occupied the position of student and teacher in the same faculty and to have worked as an engineer at a firm (I_2); in this way they will have been exposed to new *praxeologies*, that is, types of tasks, techniques, and discourses. It is also possible for an individual to have been educated (position of student) in a faculty of Mathematics (I_3) and subsequently work as a teacher (a different position) in a faculty of Engineering (I_1). These situations, among others, lead to the idea of *personal relationship*. If we define an *object* as any entity, material or immaterial, that exists for at least one individual, then every subject x has a *personal relationship* with an object o. This *personal relationship* develops as a result of the interactions that x has with o in different institutions I, where x occupies a given position p, solving tasks where o is put into play or developing discourses where o plays an important role. The *personal relationship* includes elements such as 'knowledge', 'know-how', 'conceptions', 'competencies', 'mastery', and 'mental images' (Chevallard, 1989, p.227). All subjects in a position p within I are influenced by the *institutional relationship* with o ($R_I(p, o)$). This *institutional relationship*—which is defined as the relationship with o which should ideally be that of the subjects in position p within I—remodels subjects' *personal relationship* with o. However, this may results in conflicts: a subject could have a *personal relationship* with an object that is at odds with the *institutional relationship* with that object. For instance, students entering university often have a *personal relationship* with functions, mostly crafted through their experiences in school and everyday life, which is not always compatible with the formal vision of functions they encounter in rigorous mathematics courses.

These tools allow us to model situations such as the ones that are the focus of our research. For instance, an individual who studies limits in a faculty of mathematics will develop her or his *personal relationship* with limits under the restrictions of $R_M(s, l)$. This *personal relationship* may be different than that of an individual who studies limits in a faculty of engineering and is subjected to $R_E(s, \lambda)$ (of course, it is also arguable that the position of each student, s, is different in each faculty). If these two individuals go on to teach limits in a faculty of engineering, they will be subjected to the *institutional relationship* $R_E(t, \lambda)$. This will further influence their *personal relationship,* which we conjecture has already been shaped by their different learning experiences. This situation can be more complex if the individuals work/have worked as engineers in addition to teaching in a faculty of Engineering, or if they teach/have taught in other faculties as well. We believe that ATD can offer an interesting lens through which to observe and analyse these phenomena and identify differences between teachers' *personal relationships,* which might explain their divergent practices and the various choices they make in preparing courses.

METHODOLOGY

We interviewed six university teachers with different academic backgrounds, who teach Calculus in engineering programs at two different private universities in Brazil

(A and B). All six had been teaching Calculus in Engineering for at least fifteen years. Prior to the interviews, we sent them a questionnaire to collect information on their academic and professional background, which allowed us to classify their profiles (Figure 1):

Figure 1: Profile of the six teachers

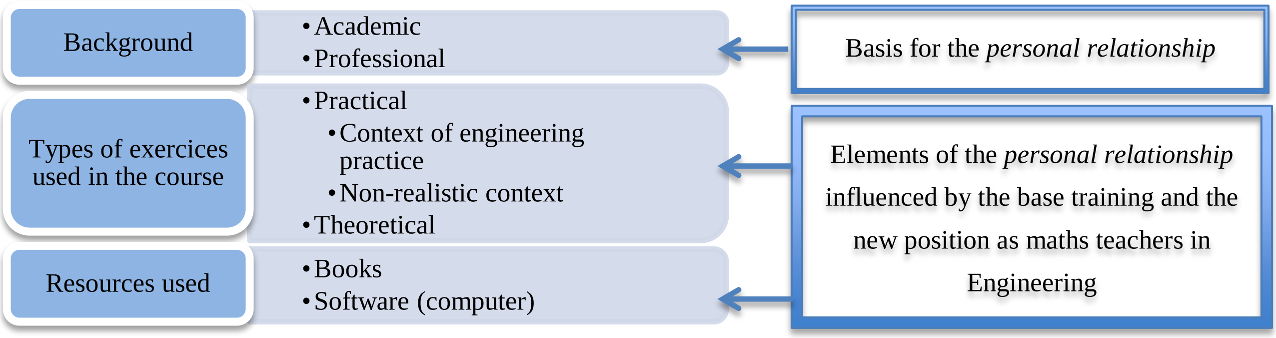
T1	T2	T3	T4	T5	T6
•Female	•Female	•Female	•Male	•Female	•Male
•Bachelor of Mathematics	•Bachelor of Mathematics	•Bachelor of Mathematics	•Bachelor of Mathematics	•Bachelor of Electrical Engineering	•Bachelor of Electrical Engineering
•Master of Mathematics Education	•Master of Mathematics	•Master of Space Eng. and Technol.	•Master of Theology	•Master of Electrical Engineering	•Master of Mathematics Education
•Doctorate of Mathematics Education	•Doctorate of Mathematics	•Doctorate of Mechanical Engineering	•None	•Doctorate of Electrical Engineering	•Doctorate of Mathematics Education

We are currently analysing data from our sample, the results of which will form the basis for future publications. For this paper, we have chosen to focus on the interviews with teachers T3 and T6. This is because T3's profile could be considered typical for teachers in engineering faculties, and because T6's background, while also typical, is augmented by postgraduate studies in postsecondary mathematics education—on topics introduced in his Calculus courses—which could explain important differences in his *personal relationship* with Calculus and its teaching.

Both instructors teach Calculus in first-year engineering courses; T3 teaches at universities A (15 years) and B (8 years), and T6 teaches only at university B (27 years). T3's entire professional career has been as a university instructor. T6, in addition to teaching, worked as an electrical engineer early in his career, spending two years as both an engineer and a university instructor before focusing on teaching exclusively. At both universities, they teach a sixth-month course entitled Calculus I. The course covers functions, limits and derivatives, and ends with rate of change and optimisation problems.

All the interviews were conducted at the teachers' workplace in a room with only the interviewer (first author of this paper) and interviewee present, on a day chosen by the interviewee. The interviews covered topics chosen to reveal the teachers' vision of Calculus notions, how they use these notions, their background, their course and exercise preparation methods, their teaching practice, and whether the professional aspirations of their students play a factor in their approach to teaching. We sought to identify aspects of the teachers' *personal relationship* with Calculus and its teaching, and pinpoint the origin of elements that influence this *personal relationship*. The interviews took place in September 2015, and were audio recorded and transcribed. After completing the transcription, we assigned codes to the answers and explanations of the teachers, allowing us to classify the data and facilitate our research. In particular, elements that could be linked to some type of 'knowledge', ‘know-how’, ‘conceptions’ and ‘mastery’, and which can be related to specific *praxeologies*, were used as indicators to guide our analyses. Figure 2 lists the elements we discuss in this paper.

Figure 2: Final categories and subcategories



DATA ANALYSIS

We asked both teachers how their academic background influences their course preparation, their choice of student exercises, and the types of books and resources they select for their course. Their first responses were transcribed as follows:

T3: In fact, I think what helps a lot is this attitude [that engineers have] toward applications. A student in the Engineering courses, he doesn’t want a lot of theory, he wants to know how he will use these concepts in practice in his life. Obviously, [he also] needs to know about the concept, where it came from […] to build his understanding, build all that... for instance, for modelling. […] Now, this heterogeneous training is not just about thinking, solving, proving, it also is effective in engineering courses, I have no doubt. […] I have this bias of an engineer, not being an engineer [because of her background in mathematics] […] I strongly believe it influenced my training. I believe that a mathematician, a pure mathematician, has a different view of mathematics, of Differential and Integral Calculus. […] Does a mathematics course have to be the same in Engineering [as it is for mathematicians]? I don’t believe it has to be as rigorous […] A mathematician who teaches Calculus, he doesn’t think about the applications. […] He’s not thinking about temperature going up or down, or about controlling an air conditioner. An engineer, he’s much more preoccupied with this.

T6: It has an influence, yes. For instance, when you're facing a problem you have to solve. What do I always say to my students? You're going to be engineers. What does an engineer do? He solves problems. [W]hat is a problem? Then, I make a drawing […] and I say: “Here you have a right triangle with sides measuring 3, 4, and 5, […] what is the area of this triangle?” Everybody [will know the answer, but if] you have a triangle, this cathetus measures 3, this hypotenuse measures 5, and I’m not giving you the measurement of the other cathetus, and I ask you the same question. […] Then, you’re going to first calculate this cathetus to get the answer. Now, how do you calculate this? Then, here you have your problem, you need to stop and think. […]And it’s the same in any other situation. You have a problem to solve. What will you do? You'll link your islands of knowledge, you'll try to articulate, try to make links among them with what you already know [and] what you don't know. […] Here, my training as an engineer carries a lot of weight at this point. Basically at this point.

There are two common elements in their responses. First, they both believe their training has influenced their vision and practices. Second, they both seek to foster critical thinking and instil in their students the 'skills an engineer needs', which may be a by-product of their Engineering training. However, T3's response more clearly reveals elements that seem to derive from her various educational experiences: her mathematics training could have shaped her view that students must 'know about the concept' – typical in mathematical *praxeologies* – and her engineering training could explain her belief in the need for real applications (she also provides specific examples, which we interpret as indicators that she has participated in *praxeologies* involving them). Paradoxically, T6 seems to favour solving problems using mathematics in a way that diverges from an engineer's daily practice; it appears his knowledge and know-how do not come from actual engineering *praxeologies*.

Differences are also apparent in the teachers' choice of resources for their course. T3 supplements the Calculus course book adopted by the department with other books. She also employs Winplot software to help students visualise notions, and acknowledges that students today have access to the Internet and its resources at home. However, T6's attitude is quite different:

T6: Then, it's like this, this is basically a course where all teachers of Calculus use this book. We *follow* the book. The idea is to follow the book. The student missed a lesson… […] he goes here in the book […] and he'll see the lesson we gave. How do I prepare my lesson? The way we follow this book […] I, specifically, use exactly the same definition that appears in the book, I write it on the blackboard, I discuss that definition with the students.

T6 added that from time to time he uses a data projector to show students graphs or approximations, but just "to make things […] more impactful visually. To make it cooler". If we consider T6's choice of resources as indicators of his conceptions and mastery, his *personal relationship* with Calculus seems to be closer to that of a student, which could explain his almost exclusive use of a single textbook. Although he is an engineer himself, the fact he worked in the field for just two years leads us to conjecture that his *personal relationship* with Calculus is derived mainly from his experience as a student, solving most tasks while relying heavily on a textbook.

When questioned about the types of exercises (practical, theoretical, problem solving) they use in their course, the teachers again revealed some interesting differences:

T3: They are more practical. […] Some problems, and when I have application problems in engineering, I think this type of exercise is quite interesting, and can illustrate how to apply that concept in an application, in [engineering].

T6: When you get to the part about […] functions and limits [it is] basically theoretical, so it goes like: Calculate the limit; find the inverse function; […] Sketch the graph […]. It's later, in derivatives [where] we can proceed to determine maxima and minima, and rate of change problems. […] They are more practical problems, with practical application. But before that, they are quite conceptual: Do this, do that. But there, from rate of change on, we have… there's an inversed cone […] being filled at a rate of some cubic meters per minute, what is the rate of change of the height in relation to time, if the height measures *x* meters?

Once again, we see that T6's *personal relationship* with Calculus seems reduced to what is presented in his textbook and evokes, as with the examples of the triangle, tasks that seem more related to a mathematician's *praxeologies* than an engineer's. He does not question engineers' need to master the basic theoretical tools regarding limits, and he does not seem to have knowledge or mastery of application problems where functions could be applied—for instance, modelling problems—which could be useful for future engineers.

When asked about specific examples of exercises that apply to engineering practices, the teachers' responses were:

T3: Let's think about the lesson on maxima and minima of a two-variable function. You can calculate the […] tangent plane to a given point, and you can exemplify this with a spherical surface, calculating the shortest distance. You can give an example of a satellite in orbit, and then you calculate the shortest distance […] from the position of the antennas. […] Then, you get to connect the theory and obviously some applications. Obviously you make some approximations […] because […] you will not consider […] all those principles that you should obviously consider in a real job or a simulation, but you use a practical example to illustrate this concept.

T6: As we are in [the first years of Engineering], you have applications [of a different type]. […] [The exercises] are generic. […] I won't… give specific applications […] [Because it's the first semester, they are [students] who aren't yet at the professional level. So, [the exercises] are more generic, everyday situations, that anyone, from any field, would be able to work on that situation or problem.

Again, we see clear differences in their *personal relationships* which seem related to different *praxeologies*. T3 showed evidence of knowledge and know-how relating mathematical content to an application in engineering. In another point in the interview, she added that when a student asks her how a given notion will be used in practice, "I may not have thought beforehand of a direct application, or maybe there's no practical example in the book I use. But when a student asks that question, I take five seconds to think and tell him: 'Look, in this situation you're going to use this. You will use it in this application'". T3 also reveals an awareness of how the notions she teaches in her Calculus course will be applied in the more advanced courses of her university's engineering program. We believe that T3's postgraduate engineering studies introduced her to specific *praxeologies* that enriched her *personal relationship* with Calculus, and that this has had an impact on her practices. Conversely, T6 states it is not possible to give examples of concrete applications in his first-year course, showing a lack of knowledge and a (likely) limited repertoire of applications for the content he teaches; this is probably due to his not having participated in specific *praxeologies* that put these notions into practice. It seems that his repertoire of applications comes solely from the textbook he uses, and that these applications are mostly mathematical and disconnected from the field of engineering. In his case, it is not possible to draw a direct line between his postgraduate training in mathematics education and his teaching practices, at least with regard to the practical needs of engineering students.

In addition to the practical application of Calculus, the interviews explored the importance that the teachers assign to theorems and demonstrations. Whereas T3 acknowledged that they help students understand a particular notion, T6 answered:

> T6: Proof? Prove the theorem? No! […] I don't do any. I don't. There are some things I usually show them, for example, the limit when x tends to zero of $\sin(x)/x$ is one. Why is this limit 1? […] Is there an analytical proof for this? There is. Will I do it for you? I won't. Why? Because there is no interest. […] There are situations like that here… easy to convince, you get a little table, with some values close to zero, you calculate the sine, you divide one by the other, and you see it gets close to one. But there are other situations where it's not as easy to convince [students]. And then, you say: "Guys, let's not worry about this, let's move on."

Once more, we see a significant difference between T6's and T3's *personal relationship* with notions of Calculus. Their positions regarding rigor are also quite different: it seems that T3's position is influenced by her background in mathematics and *praxeologies* that demand demonstrations, whereas T6's position seems to stem from his experience as an Engineering student.

Finally, we asked the teachers about their opinion and use of technology in their courses, in particular the use of computers in their Calculus courses and in the professional practice of engineers. In general, T3 seems to see computers as powerful tools when properly used, whereas T6 seems to think that computers do not provide students with meaningful benefits.

> T3: We have very powerful computational tools. But the computer doesn't do anything on its own. Who programs it? Then, you do the programming, and you have to interpret the result. Because if you do not know the concept, [imagine] you get the result with a negative volume. And you go to your boss. A negative volume? But who created the program? It was an engineer, and he didn't realise the volume cannot be negative? He only used integrals, more integrals, he used the mathematical tool, used the computer and …? And what do you get?

T6: Before going to the computer, I'd try to develop something […] build some "gadget". […] I think it gets more attention from engineering students than the computer. Computers today are just appliances. And it's fake. […] For as much as it simulates, it's simulating, it's not reality. And I think reality is more… concrete than virtual reality. […] I, as an engineer, I have a lot of this stuff. To convince me of something, that what the computer says is real... […] I think engineers are more convinced of things this way. Not with the computer. I guess.

T3 demonstrates her belief in the need to properly apply mathematical notions and results. She stresses the importance of being able to interpret results, and evokes some know-how about programming and practical engineering cases. This might come from her postgraduate training, where she engaged in *praxeologies* using computers to build neural networks and for engineering purposes. On the other hand, T6 clearly shows his scepticism towards computers, which might be due to the fact he did not use them in his professional career, or because they were not a part of his own undergraduate engineering education.

FINAL REMARKS

Our data indicates that T3 and T6, although they occupy the same position in the same institution, hold quite different *personal relationships* with the content of their Calculus courses – due to their participation in different *praxeologies* throughout their academic and professional paths – and that this has a major impact on their practices. We have more data on these two teachers in the other categories we used to construct the interviews, which could help us better understand their vision of Calculus (and their teaching methods) and pinpoint possible origins of this vision. We expect that this data, together with the data from the other four teachers, will shed light on the various phenomena that influence university teachers' visions and practices. We are also aware of the limits of our research, and the fact that teachers' *personal relationships* may be influenced by factors outside their academic and professional experience. However, the model provided by the notion of *personal relationship* could be used to analyse influences originating outside academic and professional institutions and further illuminate university teachers' practices. At this point we do not intend to account for these elements.

Our work contributes to recent research on university teachers' practices and education (Rasmussen *et al.*, 2014). We are developing tools to further our study of phenomena already identified in Hernandes Gomes & González-Martín (2015a), which seem to have an effect on students' learning (Hernandes Gomes & González-Martín, 2015b). In the context of engineering courses, these tools can be used to examine the various possible profiles of Calculus teachers, and contribute to the debate on the type of mathematics most useful for engineering students. In line with Pinto's results (2013), our data also indicate that teachers with different training and experience may teach the 'same' content in different ways; the lack of teacher training may contribute to this variety of visions and explain why university teachers seem to craft their pedagogical *praxeologies* based on knowledge, conceptions, and mental images imported from *praxeologies* present in their academic and professional experience. By analysing the complete data from interviews with all six teachers, we expect to pinpoint influential elements that can be traced to teachers' academic and professional backgrounds. This will be the focus of further research.

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