An Inquiry-Oriented Task Sequencefor Eigentheory and Diagonalization in Linear Algebra

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We present an innovative task sequence for an introductory linear algebra course that supports students' reinvention of eigentheory and diagonalization. Grounded in the instructional design theory of Realistic Mathematics Education, the task sequence builds from students' experience with linear transformations in $\mathbb{R}^2$ to introduce the idea of stretch factors and stretch directions. This is leveraged towards defining eigenvalues and eigenvectors, reinventing methods to determine them, and connecting them to change of basis and diagonalization. In the poster, we discuss the development of the task sequence and analyses of students' work on the tasks, specifically on characterizing their approaches for developing eigentheory methods.

Keywords: Linear algebra, task design, eigentheory, inquiry-oriented instruction.

INTRODUCTION

The work presented in this poster stems from a research program focused onstudent reasoning about linear algebra. It grounded in the design-based research paradigm of classroom-based teaching experiments (Cobb, 2000), which involves a cyclical process of (a) investigating student reasoning about specific mathematical concepts and (b) designing and refining tasks that honor and leverage students' ideas towards the desired learning goals (Gravemeijer, 1994; Wawro, Rasmussen, Zandieh, & Larson, 2013). One product of this research is the Inquiry-Oriented Linear Algebra (IOLA) curricular material, designed for a first course in linear algebra at the university level. In this poster, we detail the eigentheory and diagonalizationIOLA task sequence andpresent analysis of student work, specifically on Tasks 3-4.

THEORETICAL FRAMEWORK AND METHODS

Our theoretical framework for designing instructional materials draws on Realistic Mathematics Education (Freudenthal, 1991). Briefly stated, task sequences should be based on experientially real starting points; classroom activity should support student development of models-of their mathematical activity that can be used as models-for subsequent mathematical activity; and student activity, with instructor guidance, should evolve toward the reinvention of formal notions and ways of reasoning about the mathematics initially investigated.

We operationalize the notion of inquiryboth in terms of what students do and what instructors do in relation to student activity. Students learn mathematics through inquiry as they work on challenging problems that engage them in authentic mathematical practices such as symbolizing, algorithmatizing, and theoremizing (Rasmussen, Wawro, & Zandieh, 2015). Instructors engage in inquiry by listening to

student ideas, responding to student thinking, and using student thinking to advance the mathematical agenda of the classroom community (Rasmussen & Kwon, 2007).

The data presented in our poster come from classroom teaching experiments in two sections of a first course in linear algebra during Fall 2014 at a large US university. The data sources were classroom videos that captured small-group work and whole-class discussion, students' written work from class, and photos of student work on classroom whiteboards.

RESULTS

In the IOLA task sequence on eigentheory and diagonalization,Task 1 builds from students' experience with linear transformations in $\mathbb{R}^2$to introduce the idea of stretch factors and stretch directions and how these create a non-standard coordinate system for$\mathbb{R}^2$.In Task 2, students create matrices that convert between the coordinate systems and coordinate with the transformation of Task 1 to reinvent the equation$A\vec{x} = PDP^{-1}\vec{x}$. Task 3 builds from students' experience with stretch factors and directions to reinventmethods to determine eigenvalues and eigenvectors. Finally, in Task 4 students work in $\mathbb{R}^3$ to develop the characteristic equation as a solution technique and connect eigentheory to their workwith diagonalization in Tasks 1-2.

The poster highlights student work on Tasks 3-4. For instance, prior to defining eigenvector, eigenvalue, or characteristic equation, students find the "stretch factors" (i.e., eigenvalues) and "stretch directions" (i.e., eigenvectors) of a given 2x2 matrix. Common student approaches include finding eigenvectors first or eigenvalues first through solving a system of equations, and manipulating $A = PDP^{-1}$.

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