A bridge between inquiry and transmission: the study and research paths at university level

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This paper focuses on the notion of 'Study and Research Path' (SRP) proposed in the frame of the Anthropological Theory of the Didactic, as it was designed and implemented in first-year courses at university level of business and administration degrees. First, we show how SRPs can 'live' at university level, describing the conditions and constraints under which they take place in two different university institutions. Secondly, we focus on how SRPs can promote the interaction between different teaching approaches: those derived from inquiry-based models and those based on transmission pedagogies. Finally, we indicate why this interaction is essential as it enriches the students' milieu and enables the dialectics of questions and answers, both crucial elements in the evolution of the study process.

Keywords: study and research paths, research and study activities, university level, mathematical modelling, ecology.

INTRODUCTION

Educational researchers and practitioners, at all school levels and across different countries, agree on the basic principle that teaching should not just transmit knowledge to students but should also provide them with tools to question and inquire about reality. It is thus important to promote a change of the pedagogical and school paradigms, with new roles and responsibilities assigned to teachers and students, as well as new functionalities assigned to disciplinary knowledge and, in particular, to mathematics (Chevallard, 2015). In the case of mathematics, approaches like problem-based or project-based learning (PBL), inquiry-based mathematics education (IBME), have appeared with increasing frequency over the last decades in relation to mathematics and science education (Artigue & Blomhøj, 2013), supported by policy makers and curricula guidelines. However, despite the consensus on the importance of this change of paradigm, it is also apparent that any new proposal has to survive in a set of conditions and constraints that does not ensure its long-term survival, and many of said proposals end up disappearing from daily classroom activities. Therefore, to support and analyse any kind of alternative teaching proposal, researchers need reference models that allow them to describe and evaluate the impact that these innovative teaching practices have on the school system, and their relations to institutionalised practices and knowledge (within one or more disciplines). In the face of these needs, we propose the use of the epistemological and didactic model proposed by the anthropological theory of the didactic (ATD) through the notion of study and research paths (Chevallard, 2006 and 2015) in accordance with the didactic engineering research initiated by the theory of didactic situations (Barquero & Bosch, 2015). On some occasions, SRPs have been wrongly identified as 'inquiry-based proposals', as if the transmission of knowledge was not related to their internal functioning. It is therefore important to refocus the meaning of the notions 'study' also of the 'research' in the SRP proposal, and explain how the SRP arises to dialectically combine 'inquiry' with 'transmission'. This dialectics is what might ensure mathematical instruction moving towards a change of pedagogical paradigm, what Chevallard (2015) designates as the move from 'visiting works' to 'questioning the world'.

According to Winsløw, Matheron and Mercier (2013), an SRP emphasises the dialectics between 'research' (inquiry, problem solving, problem posing, etc.) and 'study' (consulting existing knowledge, attending lectures where the teacher acts as the main means to provide mathematical knowledge, etc.) that is in fact characteristic of any learning activity, even though the proportion and quality of the two elements may vary. The term 'path' emphasises the openness of the possible routes or trajectories to be followed in an effective experimentation of the SRP. That is, the starting point of an SRP should be a 'lively' question of genuine interest for the community of study, what we call a *generating question* and refer to as Q_0, that the group of students wants (or has) to answer with the help of the group of teachers. The study of Q_0 evolves and opens many other *derived questions* that appear as the starting point of new SRPs or new branches of the initial one. Elaborating answers to Q_0 has to become the main purpose of the study and an end in itself. As a result, the study of Q_0 and its derived questions Q_i leads to successive temporary answers A_i tracing out the *possible paths* to be followed in the experimentation of the SRP. In this paper we focus on two cases of SRPs that have been designed and implemented in first-year courses at university level of business administration degrees. The first research questions we aim at answering are: *under which conditions and constraints can SRPs be integrated in regular courses of mathematics at university level? How can SRPs be connected to the traditional university teaching devices?*

Besides the design and implementation of SRPs themselves, the second research question we focus on this paper is: *how to make SRPs progress in a teaching and learning situation? Why do SRPs need the interaction of more inquiry-based teaching devices others that are more based on the transmission of knowledge?* We will use two particular cases of experienced SRPs to analyse the dialectics between inquiry and transmission, emphasizing how the interaction of different didactic devices is crucial to the survival and evolution of the SRPs. Amongst other possible interactions, we assume the following premises about possible forms of integrations of SRPs:

1. When a particular mathematical organisation or praxeology has previously been introduced to students in a more 'transmissive' way (in a traditional university lecture for instance). In the ATD, the teaching of a pre-established mathematical praxeology can be described in terms of a *study and research activity* (SRA) (see Barquero &

Bosch, 2015), which is different from an SRP where the objective is not defined in advance. Thus, if the SRP starts from questioning the rationale, the necessity and use of this specific mathematical organisation, that is questioning an SRA, we could characterize this first interaction as the *generation of an SRP from the questioning of a previously developed SRA*.

2. Along the SRP development, some derived questions may appear calling for the introduction of certain mathematical pieces of knowledge (or praxeology). In this case, the starting point of the teaching and learning (sub)process is a fixed praxeology that a group of students should learn under the guidance of the teacher(s). The didactic process can be described as an *SRA originated by the questions that emerge in the SRP*.

3. A basic gesture in an SRP is to invite students, and teachers, to look for possible tools and answers outside, in the external media, which can be helpful in our study (in the sense that they contribute to provide answers). This gesture (closer to inquiry) needs to be followed by an accurate study about how to decompose and build up these external answers to be incorporated in the SRP dynamics. In this case, the main focus of this type of SRA here generated is not a fixed piece of knowledge, but the *search*, *de-* and *re-construction* of external answers and objects according to the new SRP needs. This is the most complete interaction between SRPs and SRAs.

AN SRP ABOUT THE EVOLUTION OF A SOCIAL NETWORK

General conditions and research methodology for the testing of the SRP

We focus on an SRP that was designed and implemented in 2010/11 (followed by a second implementation in 2013/14) with first-year university students of a business administration degree at the IQS-School of Management of the Universitat Ramon Llull in Barcelona (Spain). From 2006 to 2015, our research group carried out SRPs in this degree during the subject of Mathematics. On this occasion, the SRP focused on the generating question Q_0 about the *evolution of the number of users of a social network* called *Lunatic World* (Serrano & Bosch, 2011). Q_0 was divided into three sub-questions, based on the necessary tools for their resolution that were approached in each of the terms constituting the course of Mathematics. For instance, Q_0 was partially approached using discrete models and assuming independent generations of users during the first term. The second branch, developed during the second term, was then approached using functional models, so as to fit continuous function to real data. We will refer to both cases in the following section. The a priori mathematical design of this SRP is similar to the one described in Barquero, Bosch and Gascón (2013) in the case of population dynamics.

A special device called the 'mathematical modelling workshop' was introduced in the general organisation of the course. It consisted of 90-minute weekly sessions representing one third of the students' classes, and more than half of their personal work outside of the classroom. Attendance was mandatory for the students. Evaluation of the workshop was the forty per cent of the final grade of the course.

This ran parallel to the three-hour weekly lecture sessions, which included some theory sessions and problem-solving activities. The lecturer of the course was also the person responsible for the workshop, and was accompanied by the authors of the paper who acted as observers. Attendance was mandatory for the students. In the general organisation of the workshop, students worked in teams of 3 or 4 members. Once the initial question was presented, two kinds of workshop sessions were combined every week: teamwork and presentations. In the teamwork sessions, each team had to look for temporary answers to partial questions derived from Q_0 and prepare a partial report with their answers. The reports were then defended orally in the subsequent sessions by some selected working teams. A discussion followed to state what progress had been made, and to agree on how to continue the study process. During the presentation sessions, one member of the class (named the 'secretary') prepared a report containing the main points of the discussion and the new questions proposed to follow with. At the end of the term, each student had to individually write a final report on the entire study (evolution of problematic questions, work on and with different models, relationship between them, etc.). The empirical data that were collected, upon which the *analysis a posteriori* of the SRP rested, comprised the students' team and individual reports, the teacher's written description of the work carried out during each session, the worksheets given to the students and a brief questionnaire given to the students at the end of each term.

The SRA and SRP: connecting university teaching devices

The generating questions Q_0 of the SRP we focus on are, *how does the population of users of a social network evolve over time? How can we fit models to real data and use them to forecast their future evolution?* With the SRP implementation, we verified how the sequence of questions arising from Q_0 led the students and the teacher to consider most of the main contents of the entire mathematics course (see Barquero, Serrano & Serrano, 2013). In each term, various types of mathematical models were analysed: forecasting the number of users in the short and long term, considering time as a discrete variable (first-order sequences models, 1st term), the same forecast considering time as a continuous variable (differential equations, 2nd term), and the forecast in discrete time distinguishing three user groups with different privileges (models based on matrix algebra, 3rd term). However, during the SRP, these contents appeared in a very different structure from the university's traditional organization. Instead of the classical 'logic of the mathematical concepts', the workshop was more guided by the progressive appearance of the 'dynamics of questions and answers' derived from Q_0 (see Figure 1 for an SRP representation in terms of questions and answers).

To answer these questions, new media and milieu were necessary. To facilitate the necessary *enrichment of the students' milieu* along the progress of the SRP, the interplay between the lecture sessions and the workshop was crucial. The 'theory-problem' sessions had their program defined in advance. The first term was devoted to one-variable calculus (functions, their properties, derivatives, etc.), the second term focused on 2-variable functions (definition, partial derivatives, level curves, etc.) and the third term dealt with matrix algebra.

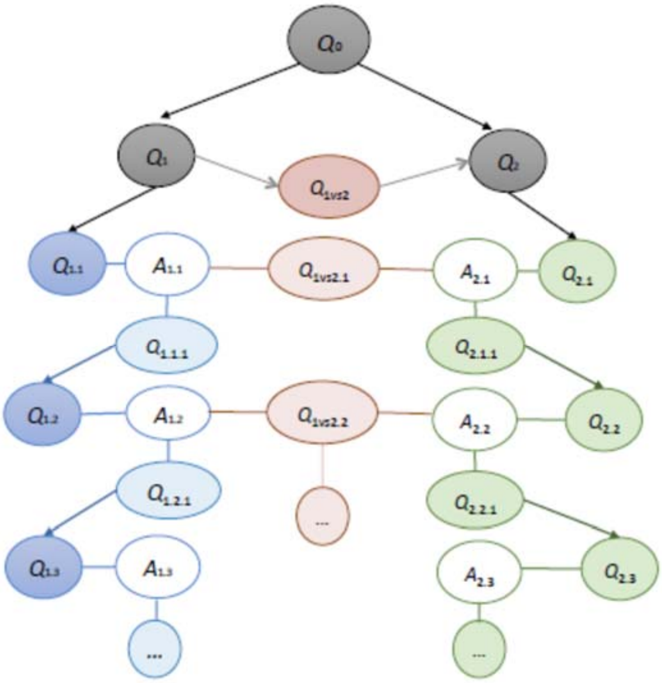


Figure 1: Question-gramme of the 1st and 2nd branch of the SRP

Q_1: If we consider time as a discrete magnitude, what assumptions about the rates of growth can we formulate? What mathematical models would appear?

$Q_{1.1}$: Assuming that the relative rate of growth is constant (p), how will the network users evolve over time? $A_{1.1}$: Construction of the Malthusian discrete model

$Q_{1.1.1}$: If the constant $p \geq 1$ (as it is the relative rate of growth average in the *Lunatic World* number of users), how can we limit the sequence modelling the network user to grow indefinitely?

$Q_{1.2}$: If we assume that the relative rate of growth decreases linearly, with K being the maximum user's capacity of the network, how will the network users evolve over time? $A_{1.2}$: Construction and study of the discrete Logistic model.

$Q_{1.2.1}$: Depending on the parameters that define the logistic model, there appear some numerical simulations that are complex to be explained (divergent, chaotic, nor-regular, etc.), why is this happening?

$Q_{1.3}$: How are these assumptions modified by considering models $x_{n+1} = f(x_n)$ where f is a C^1-function? $A_{1.3}$: Graphical simulation techniques, with f being any C^1-function.

Q_2: If we consider time as a continuous magnitude, what assumptions about the rates of growth can we formulate? What mathematical models would appear?

Q_{1vs2}: What relation can exist between the relative rate of growth and the derivative? Can the same assumptions considered in the discrete world be reformulated about the derivative?

$Q_{2.1}$: Assuming that $r(t) = p'(t)/p(t)$ is constant, how will the network users evolve over time $A_{2.1}$: Construction of the Malthusian continuous model […]

$Q_{1vs2.2}$: Do we obtain the same conclusions from the discrete and the continuous logistic model? Do the coefficients (K and α) have the same meaning and effect?

During the SRP development, there was a certain moment when questions appearing in the workshop asked for the introduction of certain mathematical tools, and the subsequent enrichment of the students' milieu to be able to follow with the study. In this case, the 'theory-problem' sessions intervened, stopping its regular running to develop a particular study and research activity (SRA) with a clear aim (the construction of a certain praxeology). More specifically, in the case of the first term (10 weeks long), three questions appeared: $Q_{1.1}$, $Q_{1.2}$ and $Q_{1.3}$ (see above) that had to carry out three particular SRA (see Figure 2, SRA 1, 2 and 4), which had not been planned in the regular course. For this reason, some of the lectures and problem sessions had to be devoted to implement the SRA to build up the necessary mathematical praxeologies about: definition of recurrent sequences and their numerical simulation with Excel (SRA1 resulting from $Q_{1.1}$, 3 hours), sequence convergence and velocity of their convergence (SRA2 resulting from $Q_{1.2}$, 1.5h) and methods of graphical numerical simulation (SRA3 resulting from $Q_{1.3}$, 2h). In these cases, the lecturer of the course acted as the main means for students, stopping the regular course and guiding the theoretical and practical activities in accordance with the SRA aims. On the other hand, some of the questions appearing in the workshop achieved to show the functionality and rationale of some contents previously introduced in the regular course. For instance, some contents of the regular course, like the study of C^1-functions, their representation, graphical techniques to solve equalities or inequalities, reappeared in the workshop, now as tools to provide answers to certain questions derived from Q_0 (as the case of $Q_{1.3}$ - $A_{1.3}$, SRA3 and SRA4 in Figure 2).

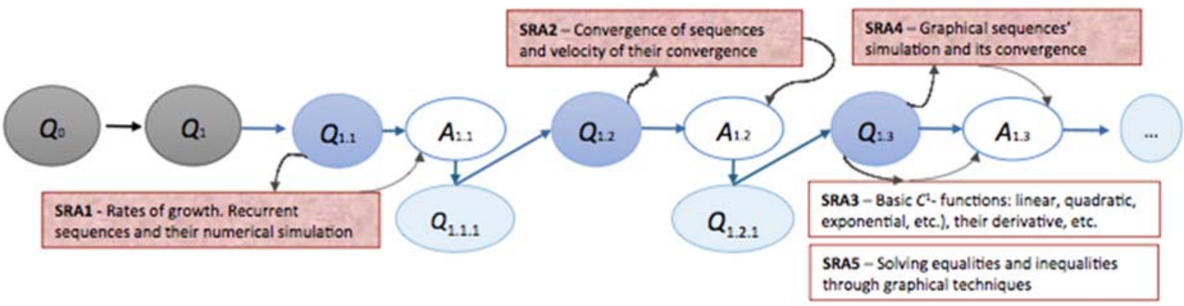


Figure 2: Interplay between the SRP and the necessary SRA during the 1st term

In the second term (also 10 weeks), the 'theory-problem' sessions were devoted to the study of multi-variable functions. In the workshop the 2nd branch of the SRP was implemented, focused on Q_2 about what continuous models can be used to fit data and to provide forecasts about the social network evolution. Students had overcome the initial resistances and progressively accepted a lot of new responsibilities they were asked to take on: defending their reports, posing new questions, looking for available answers and work outside the classroom reality, etc. Thanks to this and to the parallel structure between the 1st and 2nd branch of the SRP (see Figure 1), the students' autonomy increased significantly. When the necessity of an SRA appeared, students were first asked to search in different media (books, Internet resources, etc.) and to look for available answers to questions, getting involved in a more inquiry nature activity (see for instance Figure 3, with SRA 1 and 2 about: What is a differential equation? How can the discrete Malthusian or the logistic model be reformulated in the continuous world? Under which assumptions?). Then, some of the workshops and/or lecture sessions were used to discuss their findings according to their usefulness in the SRP.

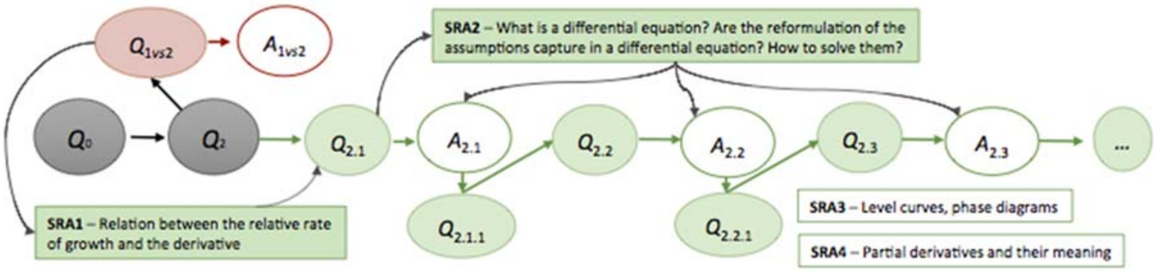


Figure 3: Interplay between the SRP and SRA during the 2nd term

AN SRP ABOUT COMPARING FORECASTS AGAINST REALITY

General conditions and the role of the SRP inside the course

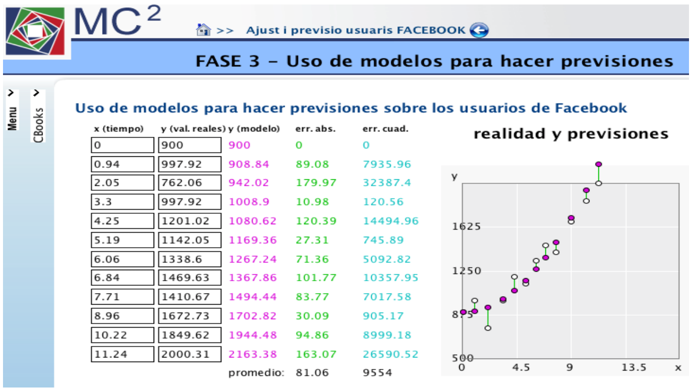
The second case we want to focus on is the case of the SRP on *comparing forecasts against reality in the case of Facebook users' evolution*. On this occasion, we only refer to the SRP a priori mathematical and didactic design, as its implementation is planned to take place in the second term of the current academic year (from January to March 2016) with first-year students of the Business Administration degree and the Marketing and Digital Communities degree at the UPF university. The design of this SRP has been carried out by the authors of this paper in the frame of the European MCSquared[1] project (http://www.mc2-project.eu).

The SRP, linked to the first-year course of Mathematics, like in the previous case, is integrated into a new teaching device called the 'modelling workshop', created for this implementation, which is offered to students as a voluntary activity outside the regular schedule of the course, adding an extra point to the final grade of the subject (if they do not fail). The workshop will run in seven sessions of 1h30 each throughout the second term, and includes a certain amount of work the students need to do outside the classroom. The workshop is planned to begin in January this year, after having worked with some important mathematical tools for this SRP: families of basic (polynomial, irrational, logarithmic and exponential) functions, notions of differential calculus and their meaning for the study of one-variable functions in the first term.

The a priori design of the SRP about the evolution of Facebook users and its integration in the institutional conditions of the university

The initial situation begins by presenting some selected journal news about a research project developed by Princeton University, which anticipated that Facebook would lose 80% of their user's before 2017 (see Figure 4). According to the forecast proposed by the Princeton research, the generating question Q_0 is presented to students as follows: *Can these forecasts be true? How can we model real data about the evolution of Facebook users and forecast the short- and long-term evolution of the social network? How can we validate Princeton conclusions?*

Figure 4: Introducing the initial question Q_0 (phase 1)



x (tiempo)	y (val. reales)	y (modelo)	err. abs.	err. cuad.
0	900	900	0	0
0.94	997.92	908.84	89.08	7935.96
2.05	762.06	942.02	179.97	32387.4
3.3	997.92	1008.9	10.98	120.56
4.25	1201.02	1080.62	120.39	14494.96
5.19	1142.05	1169.36	27.31	745.89
6.06	1338.6	1267.24	71.36	5092.82
6.84	1469.63	1367.86	101.77	10357.95
7.71	1410.67	1494.44	83.77	7017.58
8.96	1672.73	1702.82	30.09	905.17
10.22	1849.62	1944.48	94.86	8999.18
11.24	2000.31	2163.38	163.07	26590.52
		promedio:	81.06	9554

Figure 5: Use of models for the forecast (phase 3)

The workshop is composed of three interconnected phases, built up from the questions derived from Q_0 (see Figure 6). In the first phase (main focus on Q_1), the students are asked to explore and search real data about the evolution of Facebook users (from 2007 to the end of 2013, as Princeton did) and to begin with a descriptive statistical analysis of these sets of real data, their growth and tendency. In the second phase, the main question Q_2 discusses the use of models based on elementary (polynomial, exponential, logarithmic, etc.) functions to fit real data, bringing up the problem of how to better estimate the coefficients' value that define these models. Students are asked to finish this phase by proposing and justifying three mathematical models based on elementary functions. Finally, the third phase aims to use the models to forecast the short-, medium- and long-term evolution of Facebook users (Q_3) and to build up criteria to compare reality *vs* forecasts (as can be the linear or quadratic error, see Figure 5) and to describe the validity of the long-term forecasts (returning to the starting question, Q_0). Students, working in 'inquiry teams' of 3-4 people, must prepare a final report in answer to the initial question and to the derived questions that guide the three phases structuring the SRP. At certain moments of the study, the teams should prepare a partial report as a summary of their work dealing with certain questions. The discussion and debate moments, planned in advance and corresponding to the end of each phase, are crucial to ask students to defend their proposals, to decide on the new questions to face, to share new resources or answers found in the external media, etc. For instance, at the end of phase 1, a poster-presentation format was designed to help students institutionalize the real data they chose to work with and their first descriptive analysis of real Facebook data and their tendency. Discussions and debates are also to share possible complementarities among teams with respect to the real data they analyse or to the methods they propose. Moreover, it is planned that, in phase 3, students exchange their partial report at the end of phase 2 and act as reviewers and validators of another team's work.

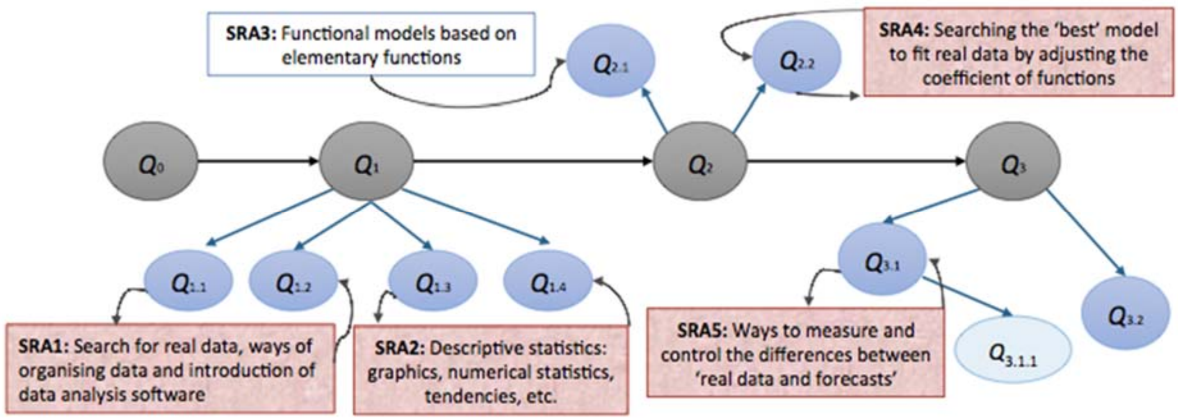


Figure 6: Question-gramme of the SRP with the necessary SRA

Concerning the necessity and interaction between the SRP progress with the SRA (see Figure 6), on the one hand, the workshop is planned after the first term when students have been introduced to one-variable functions (on a more transmission teaching model). In the SRP, however, in relation to Q_2, students will have to use functions as models to fit data, providing a new use and rationale to their previous introduction in the lectures (SRA3). On the other hand, some particular SRA will be necessary (SRA4 and SRA 5) where the lecturer participates by dedicating some regular classroom time to the introduction of these new mathematical tools. Moreover, although the workshop is mainly linked to the course of mathematics, some important contributions together with other courses have been planned. In particular, some knowledge about statistics concerning SRA1 and 2 will be necessary, running to the statistics course. A course called 'Introduction to digital communities' (starting in the 2nd term) can also provide a general sense and functionality to Q_0.

CONCLUSIONS

The notions of SRP and SRA provide a productive framework to analyse the necessary connections between 'study' activities consisting of making available a given pieces of knowledge and 'research' activities that consist of raising questions and searching, de-constructing and re-constructing answers. The two particular SRP cases presented in this paper illustrate possible ways of integrating SRA into SRP, thus linking transmissive teaching devices, like lectures or problem sessions, to more inquiry-based ones. The implementation of the second SRP will certainly shed more light on the real conditions needed and, especially, the constraints found to make these connections existent. The empirical results obtained during this experimentation will be presented at the conference.

NOTES

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REFERENCES

Artigue, M., & Blomhøj, M. (2013). Conceptualizing inquiry-based education in mathematics. *ZDM Mathematics Education, 45,* 797–810.

Barquero, B., & Bosch, M. (2015). Didactic engineering as a research methodology: from fundamental situations to study and research paths. In A. Watson, M. Ohtani (Eds.) *Task Design in Mathematics Education* (pp. 249-272). Dordrecht: Springer.

Barquero, B., Serrano, L., & Serrano, V. (2013). Creating necessary conditions for mathematical modelling at university level. In B. Ubuz, Ç. Haser & M. A. Mariotti (Eds.), *Proceedings of CERME 8* (pp. 950-960). Ankara: Middle East Tech Univ.

Barquero, B., Bosch, M., & Gascón, J. (2013). The ecological dimension in the teaching of modelling at university level. *Recherches en Didactique de Mathématiques*, 33(3), 307-338.

Bosch, M., & Gascón, J. (2010). Fundamentación antropológica de las Organizaciones Didácticas: del Taller de Prácticas Matemáticas a los Recorridos de Estudio e Investigación. In A. Bronner, M. Larguier, M. Artaud, M. Bosch, Y. Chevallard, G. Cirade & C. Ladage (Éds), *Diffuser les mathématiques (et les autres savoirs) comme outils de connaissance et d'action* (pp. 55-91).

Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. In Bosch, M. (ed.) *Proceedings of the 4th Conference of the European Society for Research in Mathematics Education* (CERME 4). (pp. 21-30) Barcelona: FUNDEMI-IQS.

Chevallard, Y. (2015). Teaching Mathematics in Tomorrow's Society: A Case for an Oncoming Counter Paradigm. In S.J. Cho (ed.), *The Proceedings of the 12th International Congress on Mathematical Education* (pp. 173-187). Dordrecht: Springer.

Serrano, V., & Bosch, M. (2011). Taller de modelización matemática: dinámica de poblaciones. *Jornadas Interuniversitarias de Innovación Docente 2011*. Barcelona: Blanquerna Tecnologia i Serveis, Universitat Ramon Llull.

Winsløw, C., Matheron, Y., & Mercier, A. (2013). Study and research courses as an epistemological model for didactics. *Educational Studies in Mathematics, 83(2)* 267-284.