Study and research paths in online teacher professional development

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We discuss a course on mathematical modelling implemented in a Master's Programme for in-service secondary mathematics teachers using an online modality. The course adopts the SRP-TE methodology proposed by Ruíz-Olarría (2015) and satisfies the principles of online participation suggested by Scott (2010). Our goal is to provide teachers with tools for creating, adapting and managing mathematical modelling activities based on the epistemological, didactic and ecological analysis proposed by the Anthropological Theory of the Didactic.

Keywords: teacher professional development, study and research paths, online education, anthropological theory of the didactic.

AN ONLINE MATHEMATICS TEACHER EDUCATION PROGRAMME

The Latin American society requires better-prepared and more professional mathematics teachers, equipped to adapt their teaching practice to the reforms and updating of new education plans. This situation is especially complex in Mexico, where no established education programme for high school or university mathematics teachers exists, so that engineers, architects and economists can directly become mathematics teachers. All these specialists turned into mathematical instructors feel a pressing need for professional development, for learning theoretical and methodological tools that will enable them to control their classrooms activities, as well as to incorporate information technologies, promote innovative forms of learning, integrate the focus on competences, etc.

While other Latin American countries have specific programs for pre-service teacher education, in-service teachers also feel the need for professional development and sometimes find it hard to obtain. Given this situation, in 2000 the *Instituto Politécnico Nacional* of Mexico created an online distance learning Master of Science Program in Mathematics Education for in-service mathematics teachers. This program, offered by the *Centro de Investigación en Ciencia Aplicada y Tecnología Avanzada* (hereinafter, CICATA-IPN) has spawned teaching units whose main objective is to disseminate the most important results of mathematics education research among the teachers. The online modality has great potential, but at the same time supposes big challenges, as shown by many investigations in general and in mathematics education. For example, Barberá and Fuentes (2012) point out that the design of teaching units must go hand in hand with the use of multimedia tools like videos, forums, *wikis*, and online correction questionnaires, among others. Costa (2009) stressed the importance of second-generation tools (e.g. web 2.0) in helping students develop their own learning paths −autonomously and in community– in more dynamic settings. Online education appears as an interesting means for teachers to transfer all the potential of information technologies into the classroom. In this respect, Scott (2010) presented empirical evidence of the efficacy of online teacher educational programs and indicates six basic principles that other authors have echoed (Castañeda & Adell, 2011):

1. Adopt a problem-solving orientation
2. Incorporate opportunities for cooperative work by teachers and with experts teachers to work together along with experts
3. Facilitate contact with innovations in knowledge, teaching practice and supporting technologies
4. Prepare teachers to test new teaching strategies and skills
5. Promote resource creation and sharing
6. Enable ongoing, purposeful reflections and discussion

In the context of mathematics teacher education, a review from Goldsmith, Doerr and Lewis of 106 studies on in-service teachers' learning from 1985 to 2008 highlight the need "to know more about the specific content of activities and, for facilitated professional learning experiences, the nature of facilitation" (Goldsmith, Doerr & Lewis, 2014, p. 22). Following these authors, we here present a research on teacher professional development working on the Anthropological Theory of the Didactic (ATD) which consists in an adaptation of the methodology of 'Study and Research Path' for teacher education (SRP-TE), initially developed by Sierra (2006) and Ruiz-Olarría (2015) for pre-service teacher education, to this online modality.

STUDY AND RESEARCH PATHS FOR TEACHER EDUCATION

The ATD stresses the need to base teacher professional development on problematic questions related to their daily practice and to particularly focus on the difficulties related to an insufficient development of the 'mathematics for teaching' (Cirade 2006). This includes problems related to the design and management of teaching and learning processes but especially to *questioning* current school mathematical organisations and the scholar knowledge they refer to. The SRP-TE methodology developed by Ruiz-Olarría (2015) takes as a starting point of the teacher educational process a teaching problematic question Q_0 proposed by the teacher-students or the educators (such as "How to teach [a given topic]?") and propose to approach it in five broad phases. The first one consists in searching for available answers Q_0 in teacher and mathematics education literature; the second one in experiencing one of these answers generated by didactics research as mathematical students (the educators acting as teachers). The third phase proposes to analyse the experience from a didactics perspective, including both an epistemological and didactic modelling of the activities carried out, which includes the introduction of analytic tools elaborated in the field of didactics. Finally, the fourth and fifth phase consist in the design, implementation (if possible) and development of a class activity adapted to the real school conditions. The SRP-TE methodology has been implemented in pre-service elementary and secondary school teacher education starting with questions related to different mathematical domains (number system, proportionality, algebra, functional modelling) and constitutes an open domain of research.

The SRP-TE methodology seems to have adapted well to the online distance learning course modality through the integration of multimedia tools like forums, videos, the *Moodle* platform, and asynchronous work conditions. Similarly, the fact that participants are in-service mathematics teachers in high schools or universities primarily in Mexico and other Latin American countries (Argentina, Chile, Colombia, Paraguay, Uruguay) offers the possibility to experiment in their classrooms or with small groups of volunteer students. This is why we decided to adapt this methodology to the design of a course initially entitled *Processes of Institutionalization of School Mathematics* (hereinafter PISM).

SRP-TE IN AN ONLINE COURSE

The main objective of the research here presented is to implement a PISM course enabling teachers to connect knowledge produced by research in mathematical education with the difficulties encountered in their teaching practice, adapting the methodology of SRP-TE to in-service teacher mathematical education. The PISM course was designed for implementation in the Masters of Science Program in Mathematical Education at the CICATA-IPN, a 2-year program that offers twelve, 4-week courses in the first three semesters. The final semester is entirely devoted to research concerning the study of a problematic issue related to the teaching practice, starting in the second semester and running parallel to the course.

Our design of the PISM course is based on four main phases: 1) experiencing an innovative teaching proposal as mathematics students; 2) analysing and adapting it for implementation in the classroom through the elaboration of a *lesson plan*; 3) putting it into practice with secondary school students; 4) identifying the institutional constraints revealed by the experimentation and subsequently redesigning the activity. These four phases are associated with the four stages of didactic analysis; *i.e.*, epistemological analysis of the content at stake; *a priori* didactic analysis; experimentation and *in vivo* didactic analysis; *a posteriori* analysis of the didactic ecology (Barquero & Bosch, 2015).

The PISM course was implemented with the CICATA *Moodle* platform and proposed four activities (one for each phase) explained in a document that also contains the program's objectives, calendar and evaluation system (see Figure 1). The online modality requires informing potential students of the entire contents from the beginning to ensure that they have a clear understanding of the nature of the theoretical and experimental activities proposed, and the requirements associated with their development. The problematic questions involved in the teaching profession that guided the design and development of this course are related to the teaching of mathematical modelling: how to analyse, adapt, develop and integrate a didactic process related to mathematical modelling into teaching practice; how to disseminate a didactic process based on modelling and ensure its long-term institutionalization; which are the difficulties to overcome; what didactic tools can help teachers to make this possible; what new questions emerge in the implementation and how to best deal with them. To approach these questions, we chose a case study on sales forecasting that was carried out and analysed with first year students of mathematics for business and management (Serrano, Bosch & Gascón, 2011). The organisation and results obtained in the four activities of the course are discussed in the following paragraphs.

Figure 1: Description of the PISM course on *Moodle*

Activity 1. Sales forecast for *Desigual*

Activity 1 presents a problem sales forecasting from *Desigual*. In this activity, students were given 11 variables representing weekly product sales in different stores in Barcelona. For example, variables 9 and 10 show the sales of one-print t-shirts in the stores situated on *Ramblas* and *Rambla de Catalunya*, respectively.

9. Evolution of weekly sales of one-print t-shirts from February 15 2010 in the *Ramblas 136* store (Barcelona):

Week	1	2	3	4	5	6	7	8	9	10	11
One-print t-shirts sold	233	112	118	130	116	151	159	173	175	230	253

10. Evolution of weekly sales of one-print t-shirts from March 1 2010 in the *Rambla de Catalunya 140* store (Barcelona):

Week	1	2	3	4	5	6	7	8	9	10
One-print t-shirts sold	100	101	107	115	125	140	140	164	194	210

In this activity, the group of teachers was divided into teams of three, and each team had to make short- and long-term forecasts for two assigned variables. They had to play the role of mathematicians working at a consulting firm –*Pi&SA*– hired by *Desigual* to forecast their sales. The instructors took on the role of senior consultants. The aim of adopting these roles is for the teachers to 'live' the modelling activity as mathematicians and experience the study of an open question to which they must provide an answer in a written report. In developing this activity we implemented a forum where participants discussed the solutions sharing files with lineal, polynomial, exponential and logarithmic models, and raised questions such as how to determine the short and long terms or how to choose the better model. They were asked to generate two reports, a partial and a final one. The objective of the partial report was to discuss their first forecasting techniques, the models used, and the means of validating them with another team. For example, team 5 presented three models –a quadratic model, an exponential one and a third one based on a modified Gompertz curve (see Figure 2)– the last one being accompanied by this analysis:

> In order to increase the fit, we considered the modified Gompertz curve, which in short periods of time enables an adequate simulation of data, and in long time periods it predicts a stationary value for the number of items sold. The disadvantage may be that it predicts a larger number of items sold in the short term than other models.

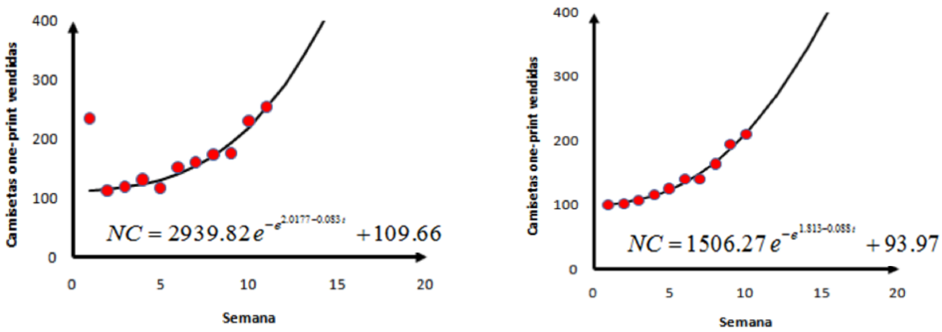

Figure 2: Modified Gompertz curve

Nº semana	Nº camisetas vendidas	Ajuste cuadrático	Ajuste exponencial	Error cuadrático	Error cuadrático al cuadrado	Error exponencial	Error exponencial al cuadrado
x_i	y_i	yc_i	ye_i	$yc_i - y_i$	$(yc_i - y_i)^2$	$ye_i - y_i$	$(ye_i - y_i)^2$
1	100	99,7857	95,9343207	-0,2143	0,04592449	-4,0656793	16,5297482
2	101	101,6428	102,725851	0,6428	0,41319184	1,72585089	2,9785613
3	107	106,5713	109,998178	-0,4287	0,18378369	2,99817755	8,98906863
4	115	114,5712	117,785338	-0,4288	0,18386944	2,78533796	7,75810755
5	125	125,6425	126,123779	0,6425	0,41280625	1,12377902	1,26287929
6	140	139,7852	135,052528	-0,2148	0,04613904	-4,94747215	24,4774807
7	140	156,9993	144,613374	16,9993	288,9762	4,61337441	21,2832235
8	164	177,2848	154,851067	13,2848	176,485911	-9,14893284	83,7029722
9	194	200,6417	165,813522	6,6417	44,1121789	-28,1864776	794,477519
10	210	227,07	177,552049	17,07	291,3849	-32,4479513	1052,86954
					802,244905		2014,32911

Figure 3: Sum of quadratic errors

In their final report, the team discussed the same three models but, inspired by the discussion with another team, added an analysis to determine which one was the most adequate. The technique they used was to select only certain data –particularly the first six– and contrast them to those provided by the company. Subsequently, they calculated the sum of the quadratic errors (see Figure 3). Once this table was obtained, they chose the quadratic model for variable 9 and the Gompertz curve for variable 10. The team argued that a larger amount of data would enable more precise forecasting. The other two teams performed activity 1 in the same way, and they all found it difficult to determine the most suitable model. Feedback from the instructors was needed to highlight the fact that the choice of one model over others had to be based on criteria that needed to be made explicit.

Activity 2. Elaborating the lesson plan

Activity 2 consisted in producing a lesson plan (or didactic guide) indicating how to implement the material on the sales forecasts for *Desigual* in a mathematics class. The aim was to let teachers adapt the open activity to the commonly assumed institutional constraints of school mathematics, so that the adaptation obtained acted as the starting point of the analyses of activities 3 and 4. Two phases were proposed to elaborate the lesson plan. The first phase was individual: participants were told to imagine that they decided to implement this activity in the classroom, but that some unforeseen circumstance obliged them to ask a colleague to take their place. They thus had to write a detailed lesson plan explaining how to conduct the activity. In the second phase, the team members discussed the individual lesson plans and had to agree on producing a collective guide. We expected that the discussions would explicitly state the institutional constraints assumed by the teachers in the design of the teaching activity. In fact, these constraints appeared even before the discussions as some teachers adopted the institutional format of the lesson plan (competences, goals, class activity, etc.) and most of them proposed a guided activity, divided into small questions or exercises where students were told on a regular basis what to do in the next step. Some of them also tried to 'open' the activity, fairly successfully. For instance, Daniela, from team 5, proposed an implementation that reflected the spirit of the modelling activity that she experienced:

> In the next sessions you'll play the role of a company that must make short- and long-term forecasts for variables that appears in the *Problem Desigual Sales.pdf* file.
>
> Activity:
>
> 1) Working individually, read the information from *Desigual* carefully and answer the following questions:
>
> a) What are the variables on which *Desigual* based the information provided? What is the nature of those variables?
>
> b) How is the variation of each variable presented?

2) Find the following information and write it down in your notebook:

a) What garments do *Desigual* stores sell?

b) What is the logo of *Desigual* stores?

c) Where are *Desigual* stores found in Spain… and outside Spain?

3) Work in teams of 3 or 4 (the instructor assigns the participants to teams)

Tasks assigned to each team:

To analyse the series assigned to it and choose the model that best represents – according to its criteria– the evolution of the variable that appears in each series. Participants are specifically asked to record in writing all their thoughts, all the decisions taken and all the agreements and disagreements experienced. They can use an *Excel* or *GeoGebra* spreadsheet to conduct their analyses.

One member of Daniela's team proposed a lesson plan in which the responsibility of the study process fell entirely on the professor, telling the students everything they needed to know in advance. In the lesson plan of the last team, it was Daniela's proposal that was adopted, with comments such as "When students solve the activities, the teacher should intervene as little as possible. His or her role should be reduced to accompanying the different teams, attentively following what they do and, if asked, pose questions that let them progress in the activity". This comment is certainly too general to be really useful…

Activity 3. Implementation of the lesson plan

Activity 3 was designed to implement the lesson plan with groups of students in ordinary classes, or with volunteer groups. Participants were asked to write a report on the implementation describing the experimental conditions, the work expected of students, an *a priori* analysis based on the team-generated lesson plan, and a description of what the students actually did. In general, participants relied on lineal, polynomic and exponential models using primarily two computer programs: *Excel* and *Geogebra*.

The teacher in charge of the implementation included this reflection in her report:

> My participation consisted in orienting the work with interventions made basically to focus the team's activity. In their eagerness to find precise mathematical responses they often chose the model that coincided with the data, refusing to consider any option that strayed from the information provided because they considered that the forecast would be unreliable. I urged them to reflect on the importance of considering models that approach reality, even if they were not so close to the company's data.

It is interesting to note how the modelling experience from activity 1 led the professors to recognize their nature and then to lead their students to experience it as well. In this case, the teacher insisted on the difficulties for her students to carry out a good 'model' activity where the conditions of the real context could be as important as the degree of precision obtained. Based on the analysis of this experimentation (phase 3 of SRP-TE) the teacher was able to identify institutional constraints associated with the traditional pedagogical paradigm, especially the passive role and lack of responsibility/ or: minimum amount of/ responsibility of the students in their learning process.

> With respect to students, the main obstacle I see is that they are not used to this dynamic. They have learned math through a traditional focus where their participation is minimal compared to the professor's activity. This difficulty can be overcome gradually as they learn to model certain problems in real contexts.

Activity 4. Joint analysis and final review of the didactic guide

The objective of the fourth and last activity was to generate a global analysis of the sales forecasting activity. To this end, each team participated in a forum where they presented and analysed the reports from the individual experimentations. They then reviewed and modified the team-generated lesson plans from activity 2 through an explicitly mathematical and didactic analysis of the activity. To elaborate this new guide, participants were given a theoretical document –produced by the educators– that presented tools for didactic analysis to describe the mathematical work developed and hence stimulate and regulate open modelling and research activities that contrast more traditional modes of transmitting knowledge. Finally, participants were asked to add a special section to compare the previously established conditions for conducting experimentations and the (individual or shared) restrictions that emerged during the implementation. Through interaction with the instructors (videos, feedback, forums), special emphasis was placed on identifying the institutional restrictions that the teachers had not foreseen, but that inevitably emerged during the experimentations.

RESULTS AND CONCLUSIONS

We have described the process of developing a course on mathematical modelling for teacher professional development based on the SRP-TE methodology. The aim of our research is twofold. On the one hand, to develop the design of SRP-TE as proposed by Ruiz-Olarría (2015); on the other hand, to adapt this methodology to online distance-learning courses for in-service teachers. With regard to the SRP-TE methodology, our research confirms the pertinence of promoting the questioning of both the mathematical school organisation of knowledge and the prevailing didactic model related to it. The teachers who carried out an open modelling activity and the way some of them 'closed' the initial problem to adapt it to the specific conditions of their teaching institution appear to be a rich *milieu* for the educational process. It helps teachers be aware of the institutional constraints affecting the implementation of new pedagogical and mathematical forms of activity. The theoretical and methodological tools of the ATD are introduced as a way to identify these constraints and to start thinking about the possible ways to overcome them (even if most of them are related to schools and pedagogical organisations of learning and are beyond the teachers' scope). We can confirm that the SRP-TE methodology can be adapted to in-service teacher education and highlight the productivity of two new devices introduced. The first one is the role-play that defines the four activities of the course (acting as a mathematician, then as a teacher-designer, then as a teacher and finally as a didactic analyst) helping teachers 'detach' themselves from their current role to look at their activity from an outsider's perspective. The second device is the lesson plan that is first prepared from the perspective of an 'ordinary teacher' in total agreement with the school institution, then implemented and finally developed after taking into account the institutional constraints assumed in its first version.

In what concerns the adaptation of the SRP-TE to the online distance-learning pedagogy, it seems to satisfy the six principles stated by Scott (2010) concerning the effectiveness of online teacher education courses. However, the online modality also presents important difficulties. The first one is related to the online mathematical activity carried out in the different phases. The teachers' interactions show a difficulty to 'speak' about the mathematical activity and, more specifically, to include parts of it in the forum. The participants' productions were always added to the forums as attached files and, thus, the exchanges could include some references to the work done, but not the pointing at some of its parts to include more specific comments ("In this calculation I used…", etc.). We do not know if any of the teams overcame this limitation by using Skype. In any case, this first analysis of the implementation of the course reveals that it is necessary to incorporate other information technology tools to improve the teachers' interactions, such as chats and, above all, those that allow direct mathematical work.

Another difficulty found was the organisation of the interactions between teachers and educators, especially with respect to the interventions of the latter in the teachers' forums. Structuring the forums threads and the most effective moments to make contributions is not clear for the educators, especially when the discussions in the teams generate several pages. This difficulty is associated with the didactic tools that should be made available to the teachers and the most appropriate moment to do so. Finally, and coming back to the initial proposal of SRP-TE by Ruiz-Olarría (2015), the PISM course does not incorporate the first phase of searching information and teaching resources available related to mathematical modelling. The way to incorporate them to the course seems difficult given the fact that the participants tend to find it too demanding in terms of work. Obviously, each new edition of the course will be an opportunity to include new developments based on didactic analyses such as the ones here presented.

NOTES

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