SRP design in an Elasticity course: the role of mathematic modelling

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We present the design process of a Study and Research Path (SRP) in a course of General Elasticity, which is part of a degree in Mechanical Engineering. General Elasticity is a field in which mathematical modelling is very much present. In fact, it cannot be conceived without mathematics. We take the observation of several didactic facts associated to the actual pedagogical and didactical organization of the course as starting point. The SRP design tries to overcome these didactical problems by proposing a possible new rationale for teaching General Elasticity.

We have carried out an a priori analysis of the SRP in order to evaluate to what extent the generating question is substantial enough to act as the main motivation of the study community. A systematic plan to collect data during experimentation is also presented.

Keywords: Mathematical modelling, Anthropological Theory of the Didactic, Study and Research Path, Engineering.

MATHEMATICS AS A SERVICE SUBJECT: MODELLING

The central point of our research is the role played by mathematics in engineering courses, more particularly those with a high load of mathematics. The paper presents the design of a Study and Research Path (SRP) in a third-year General Elasticity course of a Mechanical Engineering Degree.

In this context, the third ICMI study (Howson et al 1988) presents different reflections on mathematics as a service subject. One of the central ideas of the different papers of the study is that teaching mathematics to non-mathematicians (as a service subject) should highlight the capacity of mathematics to solve the practical problems of the domain. This "capacity to solve the problems of the domain" is closely related to the ability of mathematics to model systems of the domain and, consequently, to solve the problems associated (Romo, 2014).

The integration of mathematical modelling into current educational systems has been tackled by numerous investigations but still remains a major challenge. Many examples of mathematical modelling in various domains of engineering education exist: modelling acoustic properties of materials (Hernández, Couso, & Pintó, 2014) or the works of engineering teaching in US high schools (English & Mousoulides, 2011). Numerous theoretical approaches agree on the need to incorporate mathematical modelling in mathematics and engineering teaching in consequence. As a result, some new curricular approaches try to introduce mathematical modelling in certain some university degrees (Gould, Murray, & Sanfratello, 2012) (Dangelmayr & Kirby, 2003). Some studies consider that mathematics in engineering play such an important role that engineering could not exist without them. Because of this strong interdepence between mathematics and engineering the classical modelling cycle approach cannot be applied in this case (Bihler, Kortemeyer, & Schaper, 2015).

However, many institutional constraints and limitations appear when designing and implementing modelling devices in university teaching institutions (Barquero, Bosch, & Gascón, 2010). The institutional ecology plays a crucial role in the study of these conditions and constraints. The Anthropological Theory of the Didactic (ATD) framework enables us to describe these conditions and constraints affecting the implementation of mathematical modelling in scholar institutions, especially at university level. The necessary conditions for mathematical modelling at the undergraduate level have been studied in the case of first-year students of a business administration degree (Barquero, Serrano, & Serrano, 2013).

Mathematical modelling appears to be central in the ATD framework: the ATD postulates that "most of the mathematical activity can be identified to some extent […] with a mathematical modelling activity" (García, Gascón, Ruiz, & Bosch, 2006). This means that mathematical activity can only be understood as a collective modelling activity. The modelling activity in the ATD framework is not limited to non-mathematical systems but includes intra-mathematical modelling as a key notion. Algebraic modelling of geometry can be seen as an example of this intra-mathematical modelling.

Garcia et al. (2006) state that the modelling process emerges from an initial generating question: from this starting point the collective work will generate a collective answer to the question. This answer can be seen as a sequence of interconnected praxeologies. Intra- or extra- mathematical models play a central role: models are used to obtain results and to understand the modelled phenomena.

WHY A STUDY AND RESEARCH PATH IN AN ELASTICITY COURSE?

Beyond the mathematical role played by mathematics as a service subject and the importance of mathematical modelling, a second motivation justifies the adoption of a SRP for the General Elasticity course. Until the last academic year this course was structured in mixed theory and problem sessions, and practical sessions. The latter included six 2-hour sessions on the following topics:

- Tensile test in three different metals (AISI 304 Stainless Steel, SR 275 Structural Steel and T6061 Aluminium).
- Charpy test in three different metals (AISI 304 Stainless Steel, SR 275 Structural Steel and T6061 Aluminium).
- Finite Element Method (FEM) simulation of a tensile test (using SolidWorks™ simulation as software).
- Oral presentation about failure criteria in different family materials.

During the practical sessions in the past two academic years three didactic facts were observed. First, a *thematic autism* in the sense of Barbé et al (2005) explicitly appeared.

This means that all four activities were 'lived' as independent by the students even if the activities were closely connected. For example: FEM simulation (3rd session) simulated the real test carried out in the 1st session. The second didactic fact is related to the role played by the computer during the FEM simulation. Students introduced geometrical data, loads and meshing conditions to obtain the required results. Important difficulties appeared when they tried to understand "how the computer solved the problem" and "validating the results obtained". The students tended to validate all the results without any validating process. Both factors can be understood as a "black box" phenomenon: computer simulation is not understood by students and thus hinders them when judging the adequacy of the results obtained. And thirdly, we detected a clear absence of rationale in the four practical sessions. Both for students and for lecturers the presence of these sessions was more due to its "classical" character in elasticity than to a well-founded and justified didactic choice.

It seems that the adoption of a SRP based on a substantial enough generating question may partially overcome these limitations. The choice of the generating question emerges from the question "Why is General Elasticity taught in engineering?" which necessarily leads to the missing rationale. Once this question is posed, it is clear that the main reason to teach the subject is to provide engineers with tools enabling them to design any part of a machine working under an elastic regime. The connection between themes comes up immediately. To begin with a specific issue, the two lecturers teaching the subject agreed to start the SRP with the generating question: "How to choose one material (with unknown mechanical properties) from a set of three and design a part for a bike given in advance (brake lever, crank, gear, and bike lock key)?"

PROFESSIONAL PRACTICE FOR ENGINEERS

Some approaches~~,~~ state that mathematical education as a service subject in the engineering context should also take into account the professional practice of the collective addressed. Two studies in particular highlight the role played by mathematical modelling in the professional practice of engineers (Kent and Noss, 2003) (Gainsburg, 2006). Both studies consider mathematical modelling as a paramount in the professional practice if engineers. However Gainsburg (2006) claims some crucial aspects of mathematical modelling needed to be taken into account. Firstly, she states that the "centrality" of mathematical modelling in professional practices is not identified in all research works. Gainsburg states that this phenomenon may be caused because many studies:

> ...focus on solution-generating activity (or even, ironically, on the comparison with school-type math) has prevented these researchers from detecting and reporting other activities that might count as modelling such as describing, interpreting and explaining quantitative relationships and patterns, making predictions or developing reusable solution methods. (Gainsburg, 2006, p 6).

The second aspect highlighted by Gainsburg and Kent & Noss is that the development of mathematical models is usually carried out by mathematics specialists and that the use of these models rarely calls for advanced calculation techniques. For example, civil engineers use basic mathematics 95% of the time: multiplication, division and understanding of statistics (Kent & Noss, 2003).

A third aspect to be taken into account is the important variety of models mobilised by engineers depending on the degree of abstraction. Engineers are usually able to work with received models but in some specific cases (because of their complexity or uniqueness) the model has to be adapted to the local reality: an adaptation process is left to the engineer. In this case, two particular challenges emerge: on the one hand practitioners have to *understand the phenomenon to be modelled* (which usually remains inaccessible to the engineers) and on the other hand the *model has to be kept in track* (practitioners have to be explicitly aware of the assumptions and hypotheses of the model used).

The last aspect addressed by both studies is the preponderant role computers play in the professional practice of engineers. Both studies agree that computers have caused profound changes in the professional practice of engineers. These computer-based technologies seem to have reduced routine calculations but increased the need to solve more complicated, non-routine problems. In addition, a new phenomenon might appear: the “black box” effect we mentioned earlier. That is why only the input to the computer and the given output are explicit and the calculations done by the computer remain implicit. This implicitness makes it difficult for the users of computer simulation to question the results obtained.

GENERAL ELASTICITY COURSE

Taking into account the different aspects presented in the previous sections a SRP on general elasticity has been designed and it will be experimented in two big groups (between 20 and 25 students) in September 2015 and January 2016. One of the groups will be taught by one researcher in didactics and the other one by a teacher with no didactic training. The students will work in the mechanical laboratory during eight 2-hour sessions. The laboratory is equipped with a universal tensile test machine, a Charpy test machine, computers with simulation software and two 3D-printers. Each large group of students will be divided in groups of 4 or 3 students: each small group will have one specific part to be designed.

Each group is asked to design a specific part of a bike. At the end of the eight sessions they will be asked to write a final report addressed to a fictional “bike design company”. The report must include:

- Specific dimensions of the part including its dimensional plans,
- Estimated loads
- Justification of the choice of the material
- Estimated strains that it will suffer while being used
- The adopted safety factor for stresses and strains

- Justification of the results regarding the computer simulation and the mathematical model used
- Final cost of the whole design process calculated by using the prices in table 1. If the students decide to carry out another test that is not available in the laboratory (and in the price list) a price will be decided by the teacher as long as its adequacy is justified by the group.

The requirement of explicitness of these aspects are expected to partially "enlighten" the existing "black boxes" such as computer simulation and mathematical models.

During the first session each small group of students receives three samples of different metallic materials, whose mechanical properties are totally unknown to the students. Then students are asked to write a first partial report that will be delivered after the first week. It shall include:

- Time planning for the whole design phase
- Initial budget
- First questions that have emerged and that are planned to be solved during the following week.

After this first report, a weekly report will be generated by the students. The content of the weekly reports is intended to collect data from the dynamics of the activity. In order to collect this kind of data the proposed content was:

- An updated time planning
- The questions that the team planned to ask during the week
- A description of the tasks carried out even if obtaining wrong results
- The obtained and validated answers that they have obtained (and how) from the questions of the week and derived questions.
- New questions for the next

Test / Material	**Price**
Tensile test	75 €/specimen
Hardness test	170 €/specimen
3d printer	0,25 €/printed cm^3
Engineer	50 €/h
Computer Amortization (including software licences)	1,25 € / h
Charpy test	85 €/specimen
Specimen	5 €

Table 1: Price table for different operations

EXPECTED MOBILIZED KNOWLEDGE

As an *a priori* analysis of the SRP, we have studied what kind of knowledge is expected to emerge when the students work on the design process. As a partial representation of this mobilised knowledge a question-answer map has been used (Figure 1). This tool was already used when modelling knowledge geneses from a generating question (Winsløw, Matheron, & Mercier, 2013) (Jessen, 2104).

From a mathematical point of view, the model used (generalized Hooke's Law) in general elasticity concern the use of tensors as well as their diagonalization and the eigenvectors and eigenvalues associated. In fact, the stress level of a point of a solid is characterized by a symmetrical *stress tensor* including three normal stresses and three shear stresses. The strains in a point are also characterized by a symmetrical *strain tensor* (formed by 3 normal strains and 3 shear strains). Both tensors are related (under an elastic regime) by the generalized Hooke's Law. Diagonalization of both tensors is a crucial point for two reasons. First it is a matter of economy: the stress level of a point, when diagonalized, is described by only 3 scalars (instead of 6). Secondly, the principal stresses and strains delimit all the possible tensional states and provide the conditions for failure.

Apart from mathematical knowledge many others aspects are expected to emerge. One aspect expected to appear is related to the need to estimate loads in order to feed the computer simulation as well as to establish the safety factor for the stresses and strains obtained. A second aspect is related to the use and limitations of the computer because the students have not done any previous course on FEM simulation. A FEM course is available for students as a four year subject.

Time planning and budgeting are aspects that usually remain outside the scholar knowledge related to General Elasticity but they are very much present at the professional practice of engineers. This is the main reason why students are asked to create and update on a weekly basis and to manage a limited budget defined in the first session.

Another aspect that has been considered in this a priori analysis is the mesogenetic level, specifically the media – milieu dialectics. The SRP has been designed in order to enrich the sources of information and the validation devices used by the students. The design of a bike part will ask students to validate its shape design, the choice of the material as well as the level of acceptable strains. The devices that students will choose validate these decisions are expected to enrich the milieu of the students comparing it with the previous sessions where any validation was used further than teacher's correction.

DATA COLLECTING PLAN

In order to collect all relevant data systematically a data collecting plan has been designed. First of all, the weekly reports of each group appear as a key document to be analysed. We expect these reports to include a wide range of information from both a pedagogical and didactic level including practical aspects (such as organizational issues) to content-related aspects (such as the chronogenesis: question – answer dynamics).

As part of the plan a set of interviews will be included in the collected data. Three actors of the SRP will be interviewed: the non-researcher teacher, new students, and students retaking the subject. In fact, the opinion of the non-researcher teacher and of the students retaking the subject are particularly significant because they ‘lived’ the practical sessions when the didactical facts presented in the second section where observed.

Finally the students will fill out a survey evaluating the most difficult and easy aspects faced during the project, to what extent the project helped them integrate the different parts of the subject and which strong points and weak points they identified during the course.

EXPECTED RESULTS

The first aspect to be considered is how the expected articulation of different fields of knowledge has been reached. The main tool to evaluate this articulation will be to analyse the weekly reports and the questions and answers that will appear explicitly stated. The degree of transversality of the questions and their degree of interdependence can be used as factors to be taken into account.

A second aspect concerning to which extent the “black boxes” associated to computer simulation have been enlightened has to be measured. The empirical data that will enable the measurement of this aspect is the final report. The students are required to justify not only the computer simulation options but also the mathematical model underlying the model.

Another crucial point to be evaluated is the viability of the SRP. In this case the institutional conditions and constraints hindering (or facilitating) the development of the SRP experimented have to be studied. The nature of these conditions and constraints can be diverse: from practical aspects such as difficulties in the use of the laboratory equipment by the students to the rigidity of the time structure of the sessions (8 two-hour sessions during 4 weeks).

The expected results consist in measuring to what extent the observed problematic didactic facts will be partially overcome.

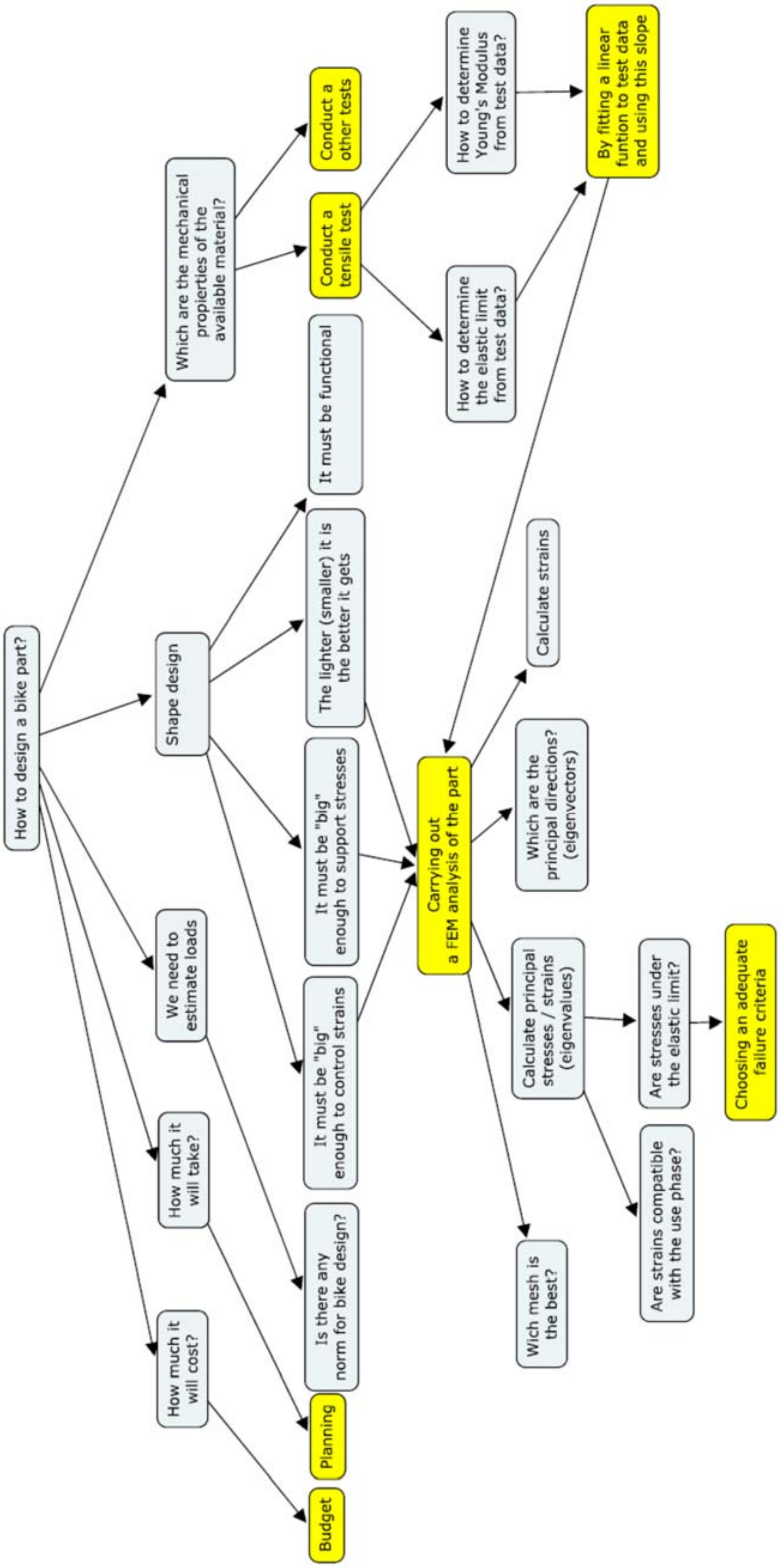

Fig 1: ***A priori*** **question-answer map**

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